Weak Value Deflection Measurement

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Introduction

Precision deflections measurements of an optical beam are useful in many fields of research. Using weak values [1], we have shown that these deflections can be amplified, which in turn improves the measurement sensitivity [2,3].

Due to photon shot noise, the signal to noise ratio (SNR) of a deflection measurement using a coherent light source scales as $\sqrt{N}$, where N is the number of photons used in the process. Thus, it is typically desirable to increase the input power (or N) in order to improve the measurement sensitivity. One is often limited, however, by detector saturation.

Using this weak value method, we have found that (1) the SNR for such measurements approaches the standard quantum limit and (2) these measurements can be made using a detector with a low saturation intensity, regardless of the input beam power.

Measurement and SNR

Position measurement:

$$\langle x \rangle = k\ell_{md}/k_0 \xrightarrow{\text{amplified}} \langle x \rangle_A = 2k\sigma^2|A_w|^2 = \alpha\langle x \rangle$$

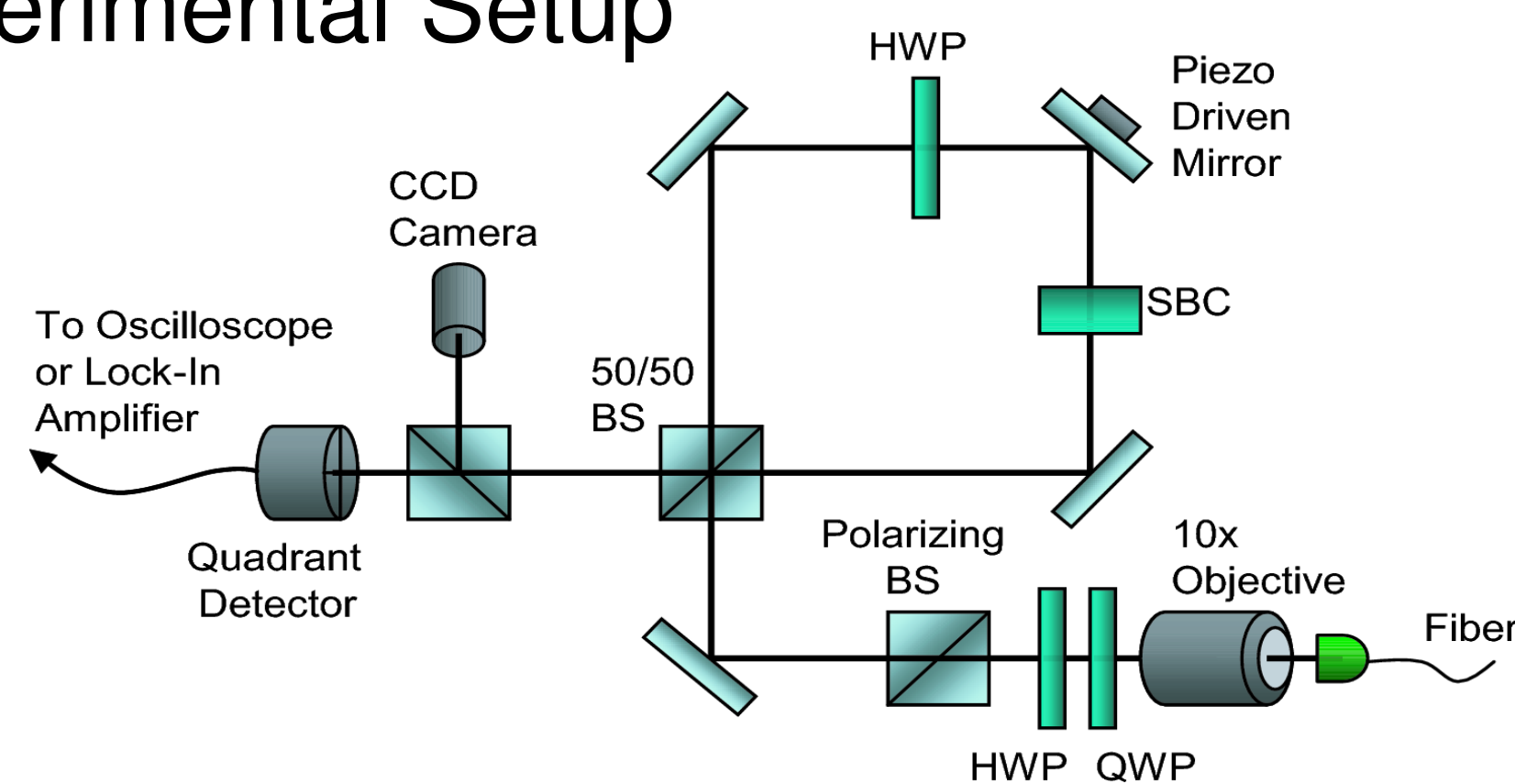amplification factor: $\alpha = 4k_0\sigma^2/\phi\, l_{md}$

SNR of measurement:

$$\mathcal{R} = \sqrt{\frac{2}{\pi}}\frac{\sqrt{N}d}{\sigma} \xrightarrow{\text{amplified}} \mathcal{R}_A = \alpha\mathcal{R}$$

amplification factor: $\alpha = 2k_0\sigma^2\cos(\phi/2)/l_{md}$

Expression with noise: $\langle x \rangle_A = \dfrac{1}{\sqrt{P_{ps}}}\left(\alpha d \pm \dfrac{\sigma}{\sqrt{\Gamma t}} \pm \dfrac{S_\xi\sqrt{P_{ps}}}{\sqrt{t}}\right)$

Definitions:

l_{md}	σ	S_ξ	Γ	k_0
distance between movable mirror (m) and detector (d)	beam radius	technical noise	photon rate	wave number

Experimental Setup

A linearly polarized coherent light source enters a Sagnac interferometer via a 50/50 beamsplitter (BS). The half-wave plate (HWP) and Soleil-Babinet compensator (SBC) adjust the interference between the two paths. The piezo driven mirror creates an opposite deflection for each path in the interferometer which is measured by the quadrant detector at the dark port. The mode quality of this port is monitored using a charge-coupled device (CCD) camera.

Results
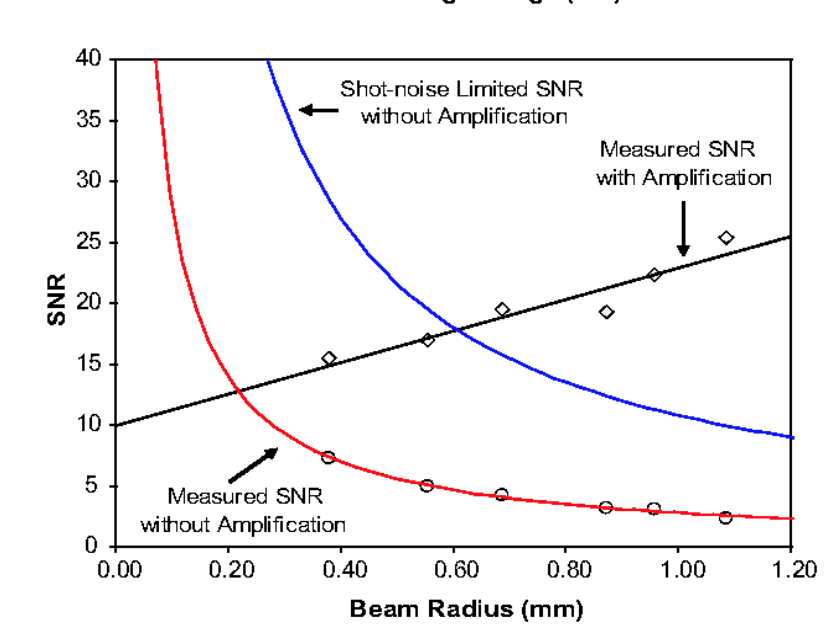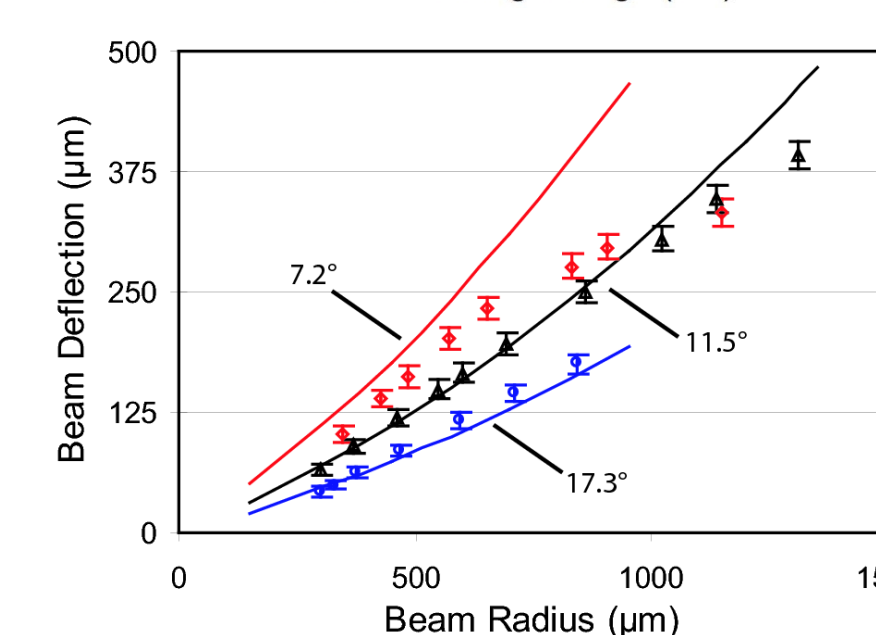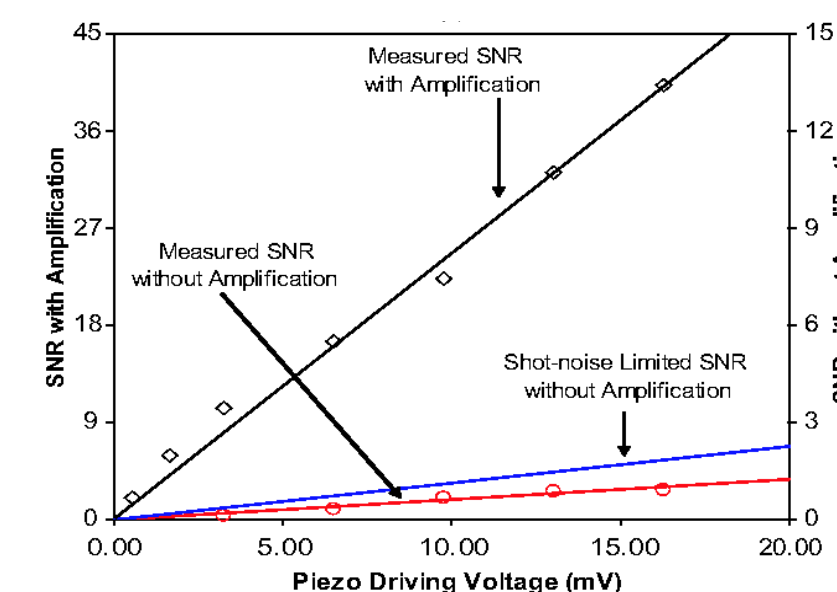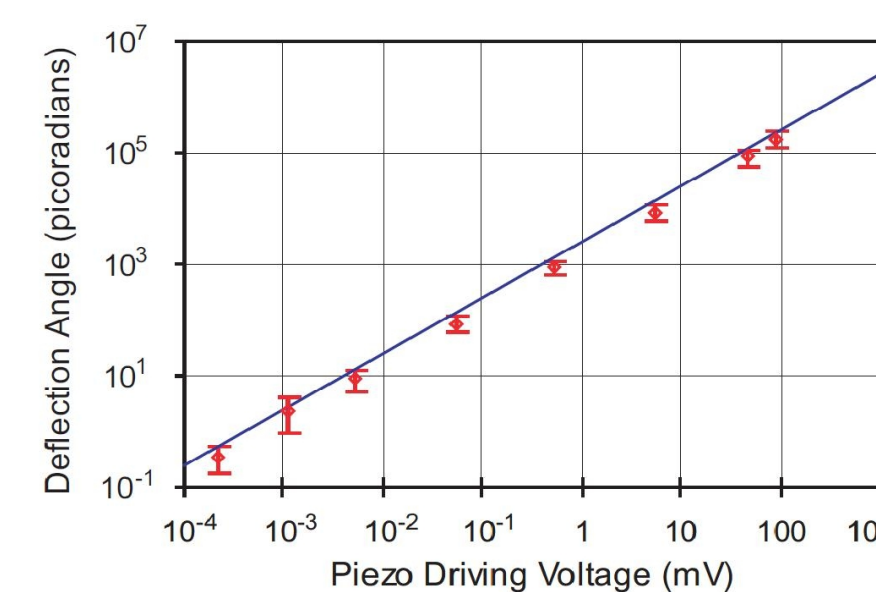
Theory (quantum treatment)

Initial Photon State: $|\psi_i\rangle = (ie^{i\phi/2}|\circlearrowright\rangle + e^{-i\phi/2}|\circlearrowleft\rangle)/\sqrt{2}$

Final Photon State: $|\psi_f\rangle = (|\circlearrowright\rangle + i|\circlearrowleft\rangle)/\sqrt{2}$

Entangled state: $|\Psi\rangle = \int dx\,\psi(x)|x\rangle\exp(-ix\mathbf{A}k)|\psi_i\rangle$

where $\mathbf{A} = |\circlearrowright\rangle\langle\circlearrowright| - |\circlearrowleft\rangle\langle\circlearrowleft|$

Post-selected state: $\langle\psi_f|\Psi\rangle = \langle\psi_f|\psi_i\rangle\int dx\,\psi(x)|x\rangle\exp(-ixA_wk)$

where the *weak value* $A_w = \dfrac{\langle\psi_f|\mathbf{A}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} \approx -2i/\phi$

Post-selection probability: $P_{ps} = |\langle\psi_f|\psi_i\rangle|^2 = \sin^2(\phi/2)$

Definitions:

k	$\{|\circlearrowright\rangle, |\circlearrowleft\rangle\}$	$\{|x\rangle\}$	ϕ	$\psi(x)$
momentum kick from mirror	system "path" states	meter "position" states	phase difference	Gaussian wavefuntion

Conclusion

Using weak value formalism we have predicted and have experimentally shown that the precision of a beam deflection measurement can be increased up to the standard quantum limit. We have demonstrated this improvement using a large beam radius and low light intensity on the quadrant detector.

References

[1] "How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100," Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. 60, 1351 (1988).

[2] "Ultrasensitive Beam Deflection Measurement via Interferometric Weak Value Amplification," P. Ben Dixon, David J. Starling, Andrew N. Jordan, and John C. Howell, Phys. Rev. Lett. 102, 173601 (2009).

[3] "Optimizing the signal-to-noise ratio of a beam-deflection measurement with interferometric weak values," David J. Starling, P. Ben Dixon, Andrew N. Jordan, and John C. Howell, Phys. Rev. A 80, 041803(R) (2009).