Extracting an entanglement signature from only classical mutual information

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Introduction

Mutual information and the Shannon entropy were first laid out in 1948 [1] and have since fueled research in many areas of classical and quantum information theory. Due to nonclassical correlations, the information capacity of quantum systems can exceed that of classical systems in some practical applications. Quantifying quantum correlations is therefore critical to understanding when and how they may be used advantageously in information processing tasks.

There exist two, classically equivalent definitions of mutual information: *I*, which is based upon joint measurements, and *J*, which is based upon conditional measurements. In the quantum framework, these two definitions are *not* equivalent. Their difference is the "quantum discord" [2] which is a measure of quantum correlations. The quantity *J* therefore represents only the classical part of the correlations between two parties.

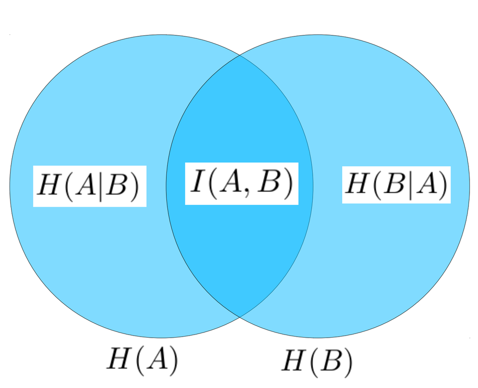
However, we show that it is still possible to extract an entanglement signature from *J*.

Shannon and von Neumann Entropy

- **Shannon entropy**
  - Is a measure of the uncertainty of a random variable
  - A random variable *A* with probability distribution *p(a)* gives
    - $H(A) = -\sum_{a \in A} p(a) \log p(a)$
  - Measured in "bits" if log is base 2
  - Evenly distributed probabilities give a higher entropy
- **von Neumann Entropy**
  - Is the quantum analog of Shannon entropy
  - A quantum state described by the density matrix ρ has von Neumann entropy
    - $S(\rho) = -\mathrm{Tr}(\rho \log \rho)$
  - Reduces to Shannon entropy upon projective measurements

Mutual Information

- Consider two random variables:
  - *A* and *B* with probability distributions *p(a)* and *p(b)*
  - The joint probability is *p(a,b)*
- The joint entropy is simply
  - $H(A,B) = -\sum_{a \in A} \sum_{b \in B} p(a,b) \log p(a,b)$
- The *conditional* entropy is
  - $H(A|B) = -\sum_{a \in A, b \in B} p(a,b) \log \frac{p(a,b)}{\sum_{a \in A} p(a,b)}$



- The mutual information is a combination of these quantities, and is a measure of how much information *A* has in common with *B*:

$$\left.\begin{array}{l} I(A,B) = H(A) + H(B) - H(A,B) \\ J(A,B) = H(A) - H(A|B) \end{array}\right\} \text{Classically Equivalent}$$

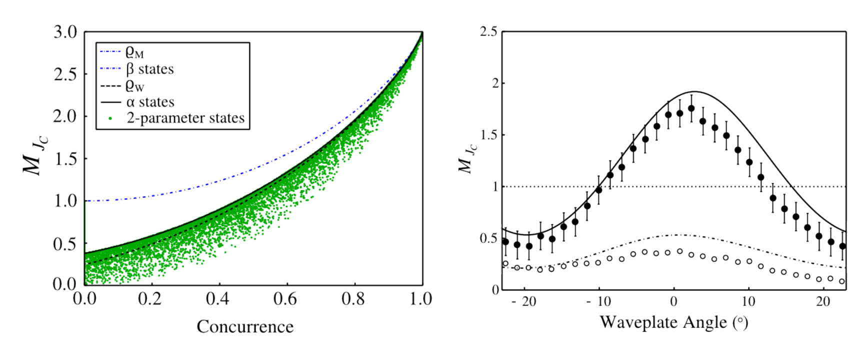
Mutual Information (quantum)

- For a quantum state ρ:
  - $I(\rho) = S(\rho^A) + S(\rho^B) - S(\rho)$
  - $J(\rho)_{\{\Pi_b^B\}} := S(\rho^A) - S(\rho|\{\Pi_b^B\})$
    - $S(\rho|\{\Pi_b^B\}) = \sum_b p(b) S(\rho_b)$ and $\rho_b = \frac{\Pi_b^B \rho \Pi_b^B}{\mathrm{Tr}[\rho \Pi_b^B]}$
    - *J* is more complicated due to the effect projective measurements has on quantum states
    - *J* represents the classical correlations, but requires maximization over all possible complete, projective measurements
  - *I* and *J* are not the same for all states
- Key examples:
  - *J* is maximal ($J = 1$) for the singlet state in *any* basis
  - *J* is maximal for the maximally correlated mixed state in *a single* basis
- What if we sum *J* measured in three mutually unbiased bases (e.g. HV, AD and RL)?

Results

- $M_{J_C} = J_C(\rho)_{\{a,b\}} + J_C(\rho)_{\{a',b'\}} + J_C(\rho)_{\{a'',b''\}}$
  - {*a,a',a''*} are, e.g., the polarization settings for qubit A, and {*b,b',b''*} for qubit B
  - The subscript *C* indicates the classical measure
  - This measure is bounded by 1 for separable states (based upon simulation)
  - Can reach 3 for maximally entangled states

Simulations | Data from Down Conversion



Hollow circles – maximally correlated mixed state (fidelity = 0.94)

Solid circles – singlet state (fidelity = 0.92)

Lines – predictions from tomographic reconstruction

Conclusion

Using classical mutual information, with local measurements on space-like separated qubits, we have shown how to construct a sufficient condition for entanglement which requires fewer measurements than a standard CHSH test [3]. We have included simulations of various states [4] and experimental data of a singlet state and a maximally correlated mixed state created from spontaneous parametric down conversion in a nonlinear crystal.

References

[1] "A mathematical theory of communication," C. E. Shannon, Bell Syst. Tech. J. 27, 379 (1949).
[2] "Quantum discord and Maxwell's demons," W. H. Zurek, Phys. Rev. A 67, 012320 (2003).
[3] "Proposed Experiment to Test Local Hidden-Variable Theories," J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969).
[4] "Comparison of the attempts of quantum discord and quantum entanglement to capture quantum correlations," A. A. Qasimi and D. F. V. James, Phys. Rev. A 83, 032101 (2011).