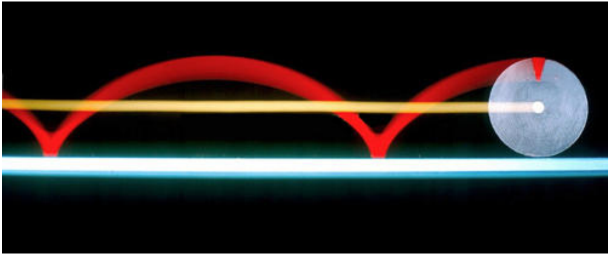
Chapter 11 - Rolling, Torque, and Angular Momentum



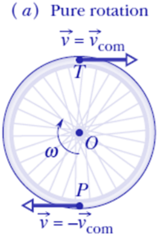
"The way to catch a knuckleball is to wait until it stops rolling and then pick it up."

-Bob Uecker

David J. Starling
Penn State Hazleton
PHYS 211

Rolling

Rolling motion is a combination of **pure rotation** *and* **pure translation**.



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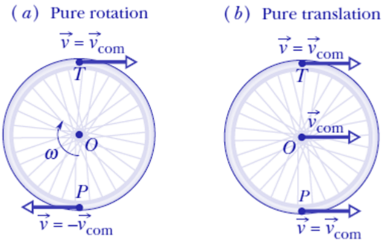

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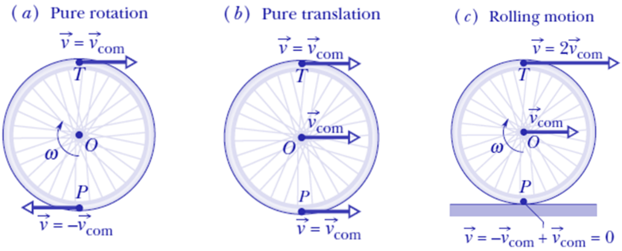

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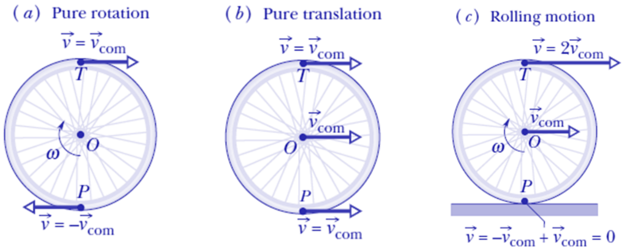
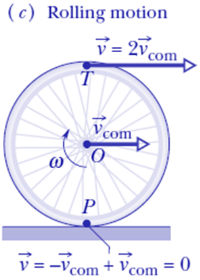


Each velocity vector on the wheel is a sum of $\vec{v}_{com}$ and the rotation.

Rolling

The relationship between the angular and linear velocities is fixed if the wheel does not slip.

$$v_{\text{com}} = R\omega$$



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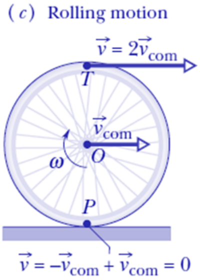
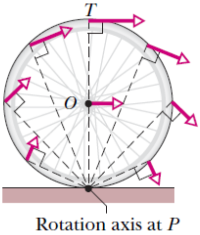


Note: therefore, $a_{\text{com}} = R\alpha$.

Rolling

If the rolling object does not slip, then the point in contact with the floor is momentarily stationary.



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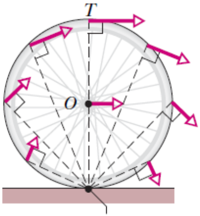
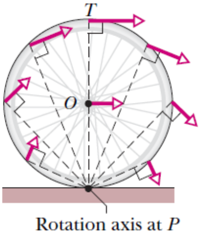


In this case, there is a static force of friction at the pivot that *does no work*.

Rolling

The kinetic energy of the wheel rotating about point P is

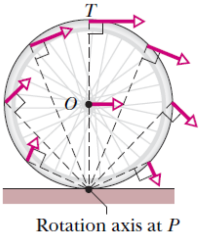
$$K = \frac{1}{2} I_p \omega^2$$



Rolling

The kinetic energy of the wheel rotating about point P is

$$\begin{aligned} K &= \frac{1}{2} I_p \omega^2 \\ &= \frac{1}{2}\left(I_{\rm com} + MR^2\right)\omega^2 \end{aligned}$$



Rolling

The kinetic energy of the wheel rotating about point P is

$$
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K &= \frac{1}{2} I_p \omega^2 \\
&= \frac{1}{2} \left(I_{\text{com}} + MR^2 \right) \omega^2 \\
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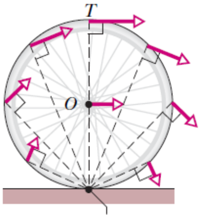
\end{aligned}
$$



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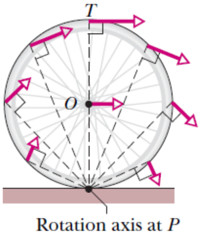

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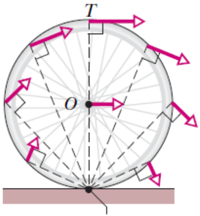
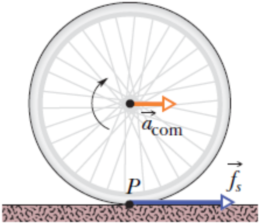
&= \frac{1}{2} I_{\text{com}}\omega^2 + \frac{1}{2} M v^2 \\
K &= K_R + K_T
\end{aligned}
$$



Rolling

When a rolling object accelerates, the friction force opposes the **tendency to slip**.



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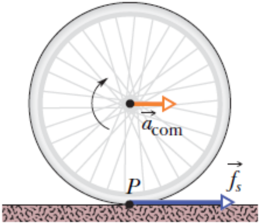


Notice how the force does not oppose the direction of motion!

Rolling

Lecture Question 11.1

Which one of the following statements concerning a wheel undergoing rolling motion is true?

(a) The angular acceleration of the wheel must be 0 m/s^2.

(b) The tangential velocity is the same for all points on the wheel.

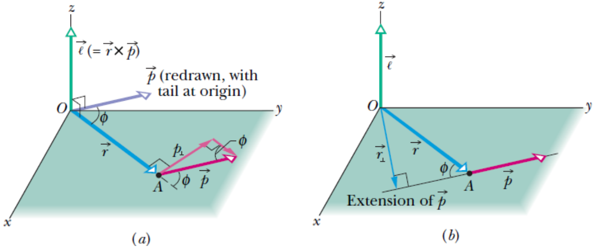
(c) The linear velocity for all points on the rim of the wheel is non-zero.

(d) The tangential velocity is the same for all points on the rim of the wheel.

(e) There is no slipping at the point where the wheel touches the surface on which it is rolling.

Angular Momentum

An object A with momentum $\vec{p}$ also has angular momentum $\vec{l} = \vec{r} \times \vec{p}$ about some point O.



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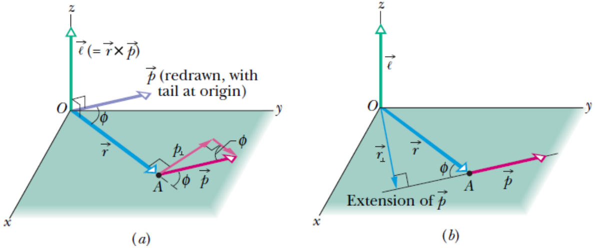

Why is it defined this way?

Angular Momentum

From Newton's 2nd Law, let's take a derivative of this "momentum":

$$\frac{d\vec{l}}{dt} = \frac{d}{dt}\left(\vec{r} \times \vec{p}\right)$$

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$$\begin{aligned} \frac{d\vec{l}}{dt} &= \frac{d}{dt}\left(\vec{r} \times \vec{p}\right) \\ &= \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} \end{aligned}$$

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&= \vec{v}\times\vec{p}+\vec{r}\times\left(m\vec{a}\right)
\end{aligned}$$

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&= \vec{r}\times\vec{F}_{net} \\
&= \vec{\tau}_{net}
\end{aligned}$$

$$\tau_{net} = \frac{d\vec{l}}{dt}$$

Angular Momentum

If there is more than one particle, the total angular momentum $\vec{L}$ is just the vector sum of the individual angular momentums $\vec{l}_i$.

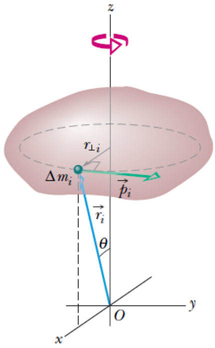
$$\vec{L} = \vec{l}_1 + \vec{l}_2 + \cdots + \vec{l}_N = \sum_{i=1}^{N} \vec{l}_i$$

And the net torque on the whole system is just:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

Angular Momentum

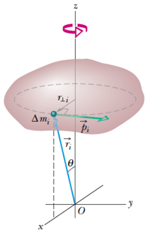
The angular momentum of a rigid body can be found by adding up the angular momentum of the particles that make it up:



(a)

$$l_i = r_{\perp i} p_i = r_{\perp i} \Delta m_i v_i$$

Angular Momentum



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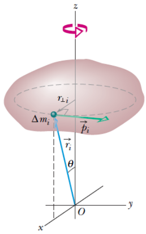
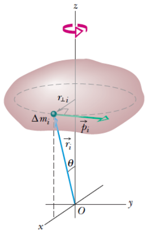


(a)

$$
\begin{aligned}
l_i &= r_{\perp i} \Delta m_i v_i \\
&= r_{\perp i} \Delta m_i (r_{\perp i} \omega)
\end{aligned}
$$

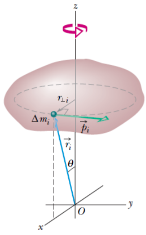
Angular Momentum



(a)

$$\begin{aligned} l_i &= r_{\perp i} \Delta m_i v_i \\ &= r_{\perp i} \Delta m_i (r_{\perp i} \omega) \\ &= (\Delta m_i r_{\perp i}^2) \omega \end{aligned}$$

Angular Momentum



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\end{aligned}
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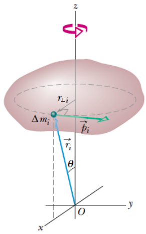
Angular Momentum



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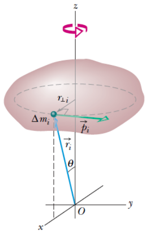
$$

Angular Momentum



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\begin{aligned}
l_i &= r_{\perp i}\Delta m_i v_i \\
&= r_{\perp i}\Delta m_i (r_{\perp i}\omega) \\
&= (\Delta m_i r_{\perp i}^2)\omega \\
L &= \sum_i (\Delta m_i r_{\perp i}^2)\omega \\
&= \left(\sum_i \Delta m_i r_{\perp i}^2\right)\omega \\
\vec{L} &= I\vec{\omega}
\end{aligned}
$$

Angular Momentum

There are many similarities between linear and angular momentum.

Translational		Rotational	
Force	$\vec{F}$	Torque	$\vec{\tau}\ (= \vec{r} \times \vec{F})$
Linear momentum	$\vec{p}$	Angular momentum	$\vec{\ell}\ (= \vec{r} \times \vec{p})$
Linear momentum[b]	$\vec{P}\ (= \Sigma \vec{p}_i)$	Angular momentum[b]	$\vec{L}\ (= \Sigma \vec{\ell}_i)$
Linear momentum[b]	$\vec{P} = M\vec{v}_{com}$	Angular momentum[c]	$L = I\omega$
Newton's second law[b]	$\vec{F}_{net} = \dfrac{d\vec{P}}{dt}$	Newton's second law[b]	$\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$
Conservation law[d]	$\vec{P}$ = a constant	Conservation law[d]	$\vec{L}$ = a constant

[a]See also Table 10-3.
[b]For systems of particles, including rigid bodies.
[c]For a rigid body about a fixed axis, with L being the component along that axis.
[d]For a closed, isolated system.

Lecture Question 11.2

What is the direction of the Earth's angular momentum as it spins about its axis?

(a) north

(b) south

(c) east

(d) west

(e) radially inward

Conservation of Angular Momentum

If the net external torque on a system is zero, the angular momentum is conserved.

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0 \rightarrow \vec{L} = constant$$

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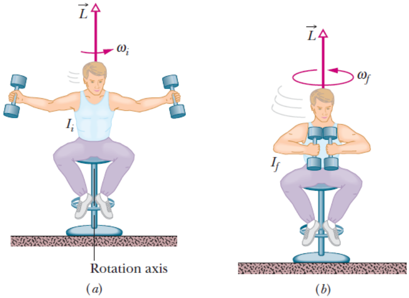

Conservation of Angular Momentum

A gyroscope is an object that spins very quickly. Such an object behaves obeys the angular Newton's Second Law.

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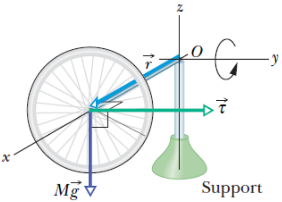

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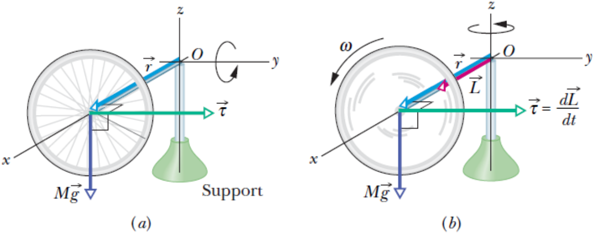
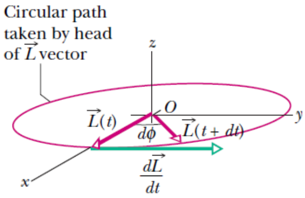


Conservation of Angular Momentum

When a gyroscope is subjected to a gravitational force it **precesses**.



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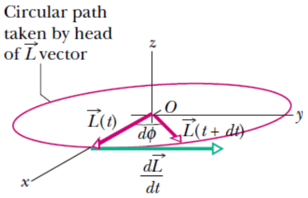

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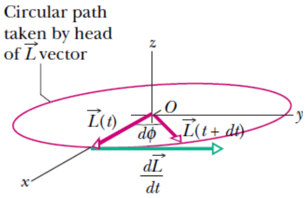

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$
$$dL = \tau\, dt = Mgr\, dt$$

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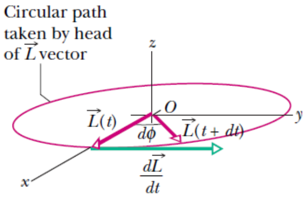

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$dL = \tau\, dt = Mgr\, dt$$

$$d\phi = \frac{dL}{L} = \frac{Mgr\, dt}{I\omega}$$

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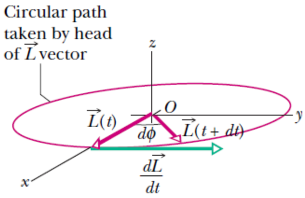

$$
\begin{aligned}
\vec{\tau} &= \frac{d\vec{L}}{dt} \\
dL &= \tau\, dt = Mgr\, dt \\
d\phi &= \frac{dL}{L} = \frac{Mgr\, dt}{I\omega} \\
\frac{d\phi}{dt} &= \frac{Mgr}{I\omega} \text{ (rate of precession)}
\end{aligned}
$$

Conservation of Angular Momentum

Lecture Question 11.3

A solid sphere of radius R rotates about an axis that is tangent to the sphere with an angular speed ω. Under the action of internal forces, the radius of the sphere increases to $2R$. What is the final angular speed of the sphere?

(a) $\omega/4$

(b) $\omega/2$

(c) ω

(d) 2ω

(e) 4ω