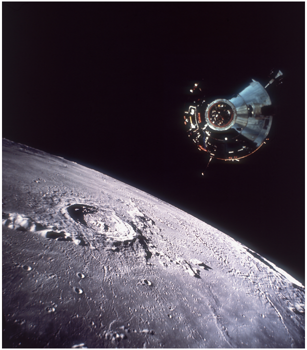
Chapter 13 - Gravity



"The moon is essentially gray, no color. It looks like plaster of Paris, like dirty beach sand with lots of footprints in it."

-James A. Lovell

(from the Apollo 13 mission)

David J. Starling

Penn State Hazleton

PHYS 211

Newton's Law of Gravitation

The gravitational force is a mutual force between two separated objects (distance r) of masses m_1 and m_2 given by

$$F = G\frac{m_1 m_2}{r^2}$$
$$G = 6.67 \times 10^{-11}\ \mathrm{m}^3/\mathrm{kg\text{-}s}^2.$$

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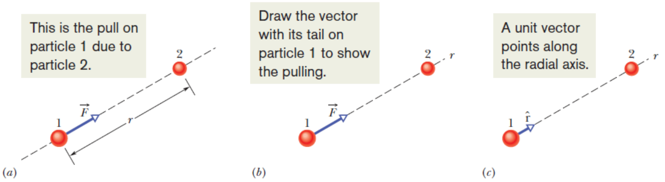
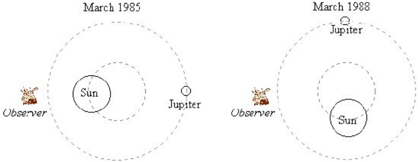
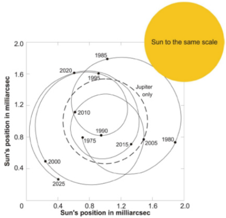


Newton’s Law of Gravitation

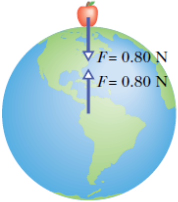
From Newton’s third law, we know that this force must have an equal but opposite pair.





Newton’s Law of Gravitation

Shell Theorem: a uniform sphere of matter attracts a particle that is outside as if all the sphere’s mass were concentrated at its center.



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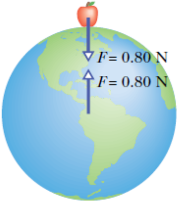


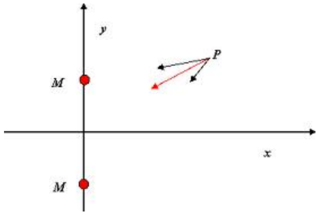
Uniform spherical objects just become points.

Newton's Law of Gravitation

Principle of Superposition: *If N objects interact with particle 1 gravitationally, the total force is just the vector sum.*

$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \cdots + \vec{F}_{1N}$$

$$\vec{F}_{1,net} = \sum_{i=2}^{N} \vec{F}_{1i}$$



Newton's Law of Gravitation

We can apply Newton's Law of Gravitation to an object (m) near the surface of the Earth (M):

$$\vec{F} = G\frac{Mm}{r^2} = ma_g$$

$$a_g = \frac{GM}{r^2}$$

Table 13-1

Variation of a_g with Altitude

Altitude (km)	a_g (m/s^2)	Altitude Example
0	9.83	Mean Earth surface
8.8	9.80	Mt. Everest
36.6	9.71	Highest crewed balloon
400	8.70	Space shuttle orbit
35 700	0.225	Communications satellite

Newton's Law of Gravitation

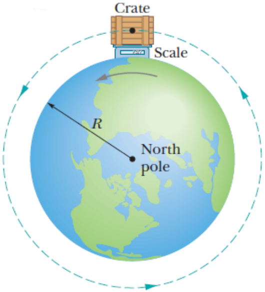
The force due to gravity on the surface of the earth is not consistently 9.83 m/s^2.

- Earth is not a perfect sphere;
- the mass within the Earth is not uniformly distributed;

Newton's Law of Gravitation

The force due to gravity on the surface of the earth is not consistently 9.83 m/s^2.

- Earth is not a perfect sphere;
- the mass within the Earth is not uniformly distributed;
- Earth *rotates*.



Newton's Law of Gravitation

Lecture Question 13.1

If an object at the surface of the Earth has a weight W, what would be the weight of the object if it was transported to the surface of a planet that is one-sixth the mass of Earth and has a radius one third that of Earth?

(a) $3W$

(b) $4W/3$

(c) W

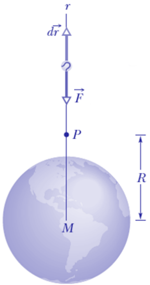
(d) $3W/2$

(e) $W/3$

Gravitational Potential Energy

Gravity is a conservative force, so lets find its potential energy using $\Delta U = -W$.

$$\Delta U \quad = \quad -W$$



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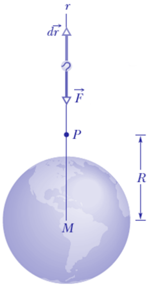

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\begin{aligned}
\Delta U &= -W \\
U(\infty) - U(R) &= -\int_R^{\infty} \vec{F}(r) \cdot d\vec{r}
\end{aligned}
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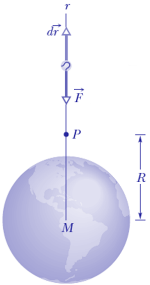

$$\begin{aligned} \Delta U &= -W \\ U(\infty) - U(R) &= -\int_R^\infty \vec{F}(r) \cdot d\vec{r} \\ &= -GMm \int_R^\infty \frac{1}{r^2} dr \end{aligned}$$

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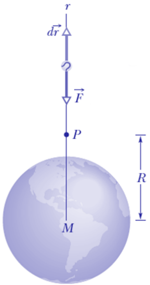

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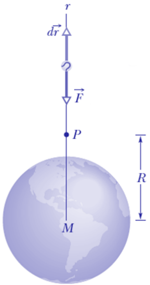

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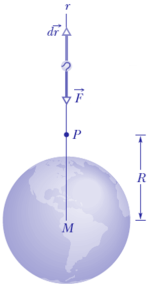

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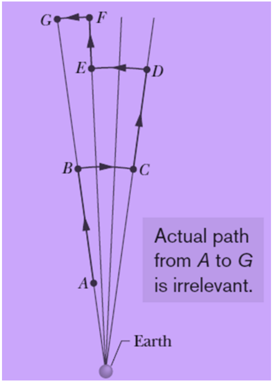
Gravitational Potential Energy

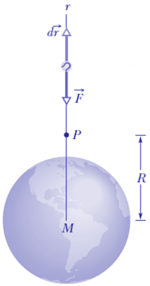
The change in gravitational potential energy ΔU is path independent.



$$\Delta U = -W$$

Gravitational Potential Energy

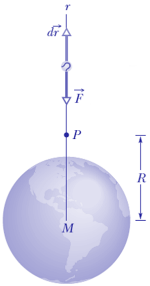
The force from this potential energy is just the derivative (since we used an integral to derive it).



$$
\begin{aligned}
F(r) &= -\frac{dU}{dr} \\
&= -\frac{d}{dr}\left(-\frac{GMm}{r}\right) \\
&= -\frac{GMm}{r^2}
\end{aligned}
$$

Gravitational Potential Energy

The force from this potential energy is just the derivative (since we used an integral to derive it).



$$\begin{aligned} F(r) &= -\frac{dU}{dr} \\ &= -\frac{d}{dr}\left(-\frac{GMm}{r}\right) \\ &= -\frac{GMm}{r^2} \end{aligned}$$

The minus sign indicates the force points radially inward.

Gravitational Potential Energy

Table 13-2

Some Escape Speeds

Body	Mass (kg)	Radius (m)	Escape Speed (km/s)
Ceres[a]	1.17×10^{21}	3.8×10^{5}	0.64
Earth's moon[a]	7.36×10^{22}	1.74×10^{6}	2.38
Earth	5.98×10^{24}	6.37×10^{6}	11.2
Jupiter	1.90×10^{27}	7.15×10^{7}	59.5
Sun	1.99×10^{30}	6.96×10^{8}	618
Sirius B[b]	2×10^{30}	1×10^{7}	5200
Neutron star[c]	2×10^{30}	1×10^{4}	2×10^{5}

[a]The most massive of the asteroids.
[b]A *white dwarf* (a star in a final stage of evolution) that is a companion of the bright star Sirius.
[c]The collapsed core of a star that remains after that star has exploded in a *supernova* event.

Gravitational Potential Energy

Lecture Question 13.2

A large asteroid collides with a planet of mass m orbiting a star of mass M at a distance r. As a result of the collision, the planet is knocked out of its orbit, such that it leaves the solar system. Which of the following expressions gives the minimum amount of energy that the planet must receive in the collision to be removed from the solar system?

(a) GMm/r

(b) GMm/r^2

(c) $GMm/\sqrt{r}$

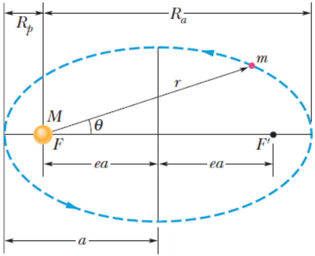
(d) Gm/r

(e) Gm/r^2

Kepler's Laws

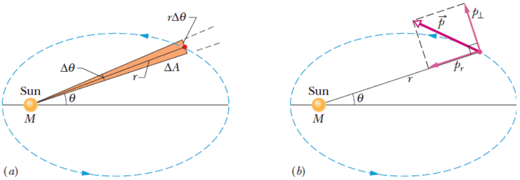
Johannes Kepler was a 17th century mathematician who developed three laws of planetary motion.

1. The Law of Orbits: all planets move in elliptical orbits with the Sun at one focus.



Kepler's Laws

2. The Law of Areas: a line that connects a planet to the Sun sweeps out equals areas in equal time intervals (i.e., $dA/dt = constant$).



$$\Delta A = \frac{1}{2}r^2(\Delta\theta) \rightarrow dA = \frac{1}{2}r^2 d\theta$$

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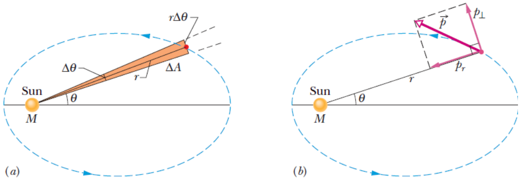

$$\Delta A = \frac{1}{2}r^2(\Delta\theta) \rightarrow dA = \frac{1}{2}r^2 d\theta$$

$$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega$$

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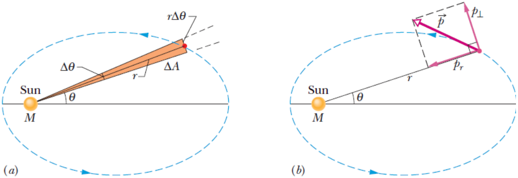

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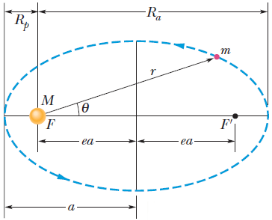
$$L = r p_\perp = r m v_\perp = r m r \omega \rightarrow \frac{dA}{dt} = \frac{L}{2m}$$

Kepler's Laws

3. The Law of Periods: the square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.



$$F = ma$$

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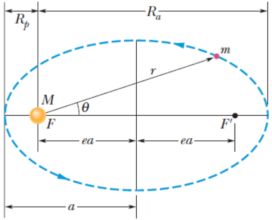

$$\begin{aligned} F &= ma \\ \frac{GMm}{r^2} &= m(r\omega^2) = mr^2\left(\frac{2\pi}{T}\right)^2 \end{aligned}$$

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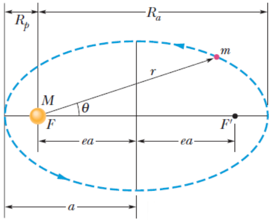

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\begin{aligned}
F &= ma \\
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T^2 &= \left(\frac{4\pi^2}{GM}\right) r^3
\end{aligned}
$$

Kepler's Laws

Table 13-3

Kepler's Law of Periods for the Solar System

Planet	Semimajor Axis a (10^{10} m)	Period T (y)	T^2/a^3 (10^{-34} y^2/m^3)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

Kepler's Laws

When one object orbits a much larger object, mechanical energy is conserved.

For a circular orbit,

$$F = ma \rightarrow \frac{GMm}{r^2} = m\frac{v^2}{r}$$

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$$F = ma \rightarrow \frac{GMm}{r^2} = m\frac{v^2}{r} \rightarrow \frac{GMm}{2r} = \frac{1}{2}mv^2 = K$$

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$$F = ma \rightarrow \frac{GMm}{r^2} = m\frac{v^2}{r} \rightarrow \frac{GMm}{2r} = \frac{1}{2}mv^2 = K$$

$$\begin{aligned} E &= K + U \\ &= \frac{GMm}{2r} - \frac{GMm}{r} \\ E &= -\frac{GMm}{2r} \end{aligned}$$

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(for an elliptical orbit, $E = -GMm/2a$)

Kepler’s Laws

The total energy of an orbiting body is negative.

$$E = -\frac{GMm}{2r}$$