Chapter 16 - Waves

"I'm surfing the giant life wave."
-William Shatner

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PHYS 213

The Basics of Waves

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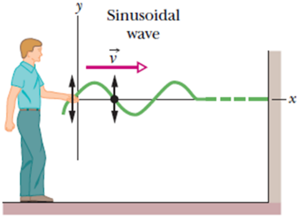
(c) Matter waves: described my quantum mechanics, these waves explain the wave nature of fundamental particles (electrons, protons, etc).

The Basics of Waves

Mechanical waves come in two flavors:



transverse

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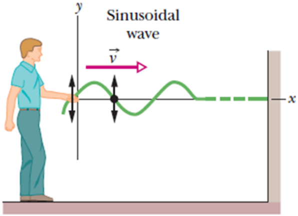
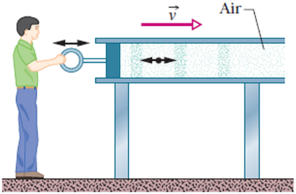


transverse



longitudinal

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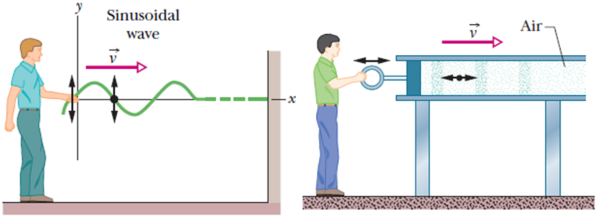

transverse **longitudinal**

The oscillations of matter are perpendicular or parrallel to the motion of the wave.

The Basics of Waves

All waves satisfy the so-called "wave equation."

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

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We are looking for $y(x, t)$.

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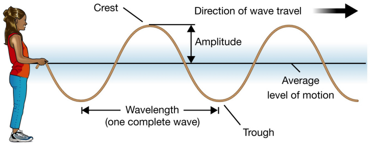
Mechanical waves are described by a traveling sine wave.

$$y(x,t) = y_m \sin(kx - \omega t)$$



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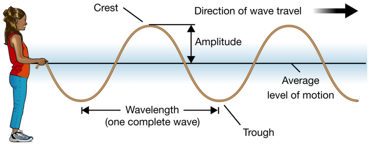
$$y(x,t) = y_m \sin(kx - \omega t)$$

- y_m: amplitude (maximum displacement)
- $k = 2\pi/\lambda$: wavenumber
- λ: wavelength
- $\omega = 2\pi f$: angular frequency



The Basics of Waves

Reminder: frequency, period and angular frequency are all related:

$$\omega = 2\pi f$$
$$f = \frac{1}{T}$$
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$$\begin{aligned} \omega &= 2\pi f \\ f &= \frac{1}{T} \\ \omega &= \frac{2\pi}{T} \end{aligned}$$

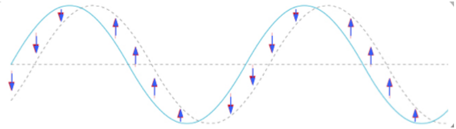
The wave number $k = 2\pi/\lambda$ is a "spatial frequency":

$$y(x, t = 0) = y_m \sin(kx + 0).$$

The Basics of Waves

Each part of the rope is confined to its x position and travels up and down like a harmonic oscillator:

$$y(x = 0, t) = y_m \sin(-\omega t)$$



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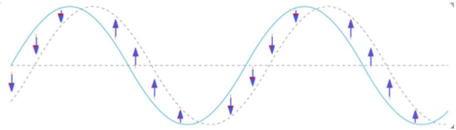
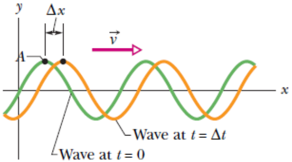


The speed of this part of the rope is just

$$u = \frac{dy(t)}{dt} = -\omega y_m \cos(\omega t)$$

The Basics of Waves

We can find the **speed of the wave** *from the argument of the sine function* $\sin(kx - \omega t)$*, known as the* **phase of the wave**.



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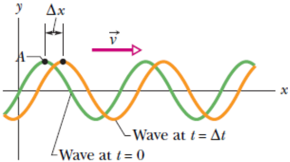

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\begin{aligned}
kx - \omega t &= constant \\
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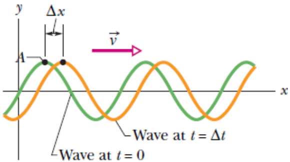
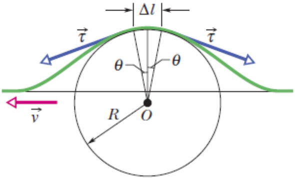


$$
\begin{aligned}
kx - \omega t &= constant \\
\frac{d}{dt}(kx - \omega t) &= 0 \\
k\frac{dx}{dt} - \omega &= 0 \\
\frac{dx}{dt} &= v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f
\end{aligned}
$$

The Basics of Waves

We can derive the wave speed for a stretched rope with tension τ and mass density μ kg/m.



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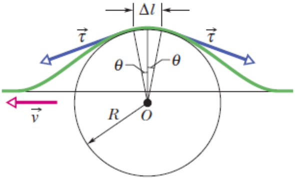

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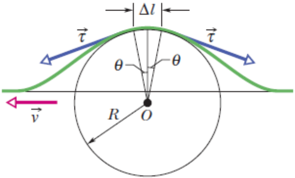

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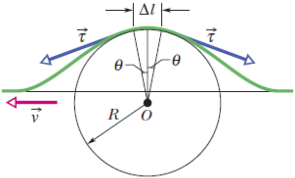

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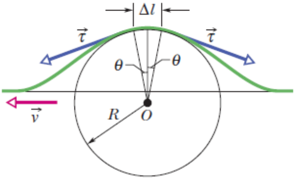

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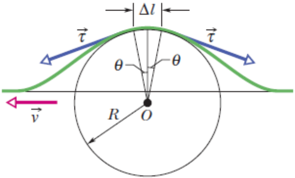

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The Basics of Waves

Lecture Question 16.1

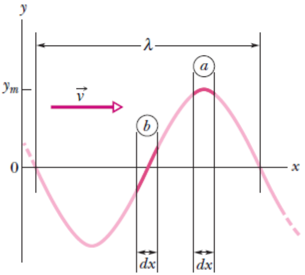
Alice and Bob are floating on a quiet river. At one point, they are 5.0 m apart when a speed boat passes. After the boat passes, they begin bobbing up and down at a frequency of 0.25 Hz. Just as Alice reaches her highest level, Bob is at his lowest level. As it happens, they are always within one wavelength. What is the speed of these waves?

(a) 1.3 m/s
(b) 2.5 m/s
(c) 3.8 m/s
(d) 5.0 m/s
(e) 7.5 m/s

Energy of Waves

Waves transfer energy in the direction of travel.

The rate of energy transfer is the power.



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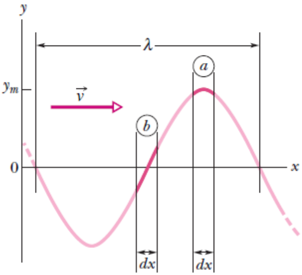

- The mass in region *b* has K but no U.
- The mass in region *a* has U but no K.

Energy of Waves

$$P_{avg} = \left(\frac{dK}{dt}\right)_{avg} + \left(\frac{dU}{dt}\right)_{avg}$$

Energy of Waves

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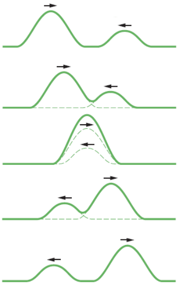
\end{aligned}
$$

The wave generates power in proportion to its mass, velocity and the square of the frequency and amplitude.

Interference of Waves

When two waves on a string overlap, their displacements add algebraically resulting in **interference***:*

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$



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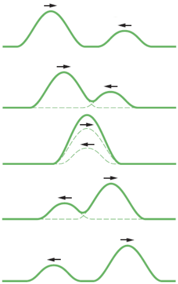

Note: the waves emerge without alteration.

Interference of Waves

Special case: two waves of equal magnitude, wavelength and velocity travel along the same string.

$$
\begin{aligned}
y'(x,t) &= y_1(x,t) + y_2(x,t) \\
&= y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi)
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&= y_m[\sin(kx - \omega t + \phi/2 - \phi/2) \\
&\quad + \sin(kx - \omega t + \phi/2 + \phi/2)]
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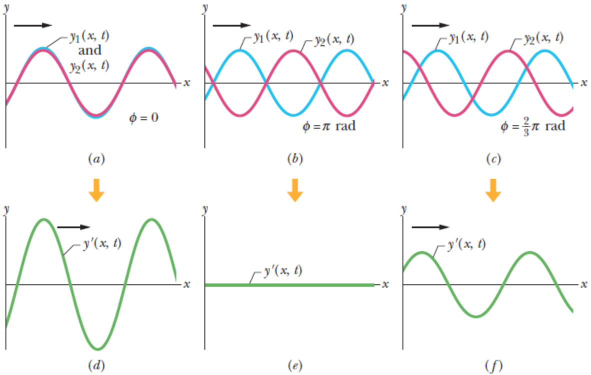
Interference of Waves

$$y'(x,t) = [2y_m \cos(\phi/2)] \sin(kx - \omega t + \phi/2)$$



Interference of Waves

Table 16-1

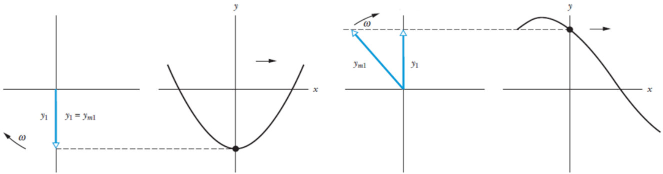
Phase Difference and Resulting Interference Types[a]

Phase Difference, in			Amplitude of Resultant Wave	Type of Interference
Degrees	Radians	Wavelengths		
0	0	0	$2y_m$	Fully constructive
120	$\frac{2}{3}\pi$	0.33	y_m	Intermediate
180	π	0.50	0	Fully destructive
240	$\frac{4}{3}\pi$	0.67	y_m	Intermediate
360	2π	1.00	$2y_m$	Fully constructive
865	15.1	2.40	$0.60y_m$	Intermediate

[a]The phase difference is between two otherwise identical waves, with amplitude y_m, moving in the same direction.

Interference of Waves

We can represent a wave with a **phasor**, *a vector of length* y_m *that rotates about the origin at a frequency of* ω.



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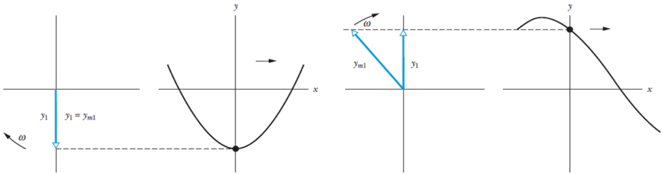
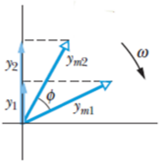
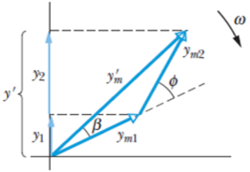


The vertical projection is the displacement of the wave at a particular point.

Interference of Waves

For two waves, you add the phasors to get the resulting displacement.

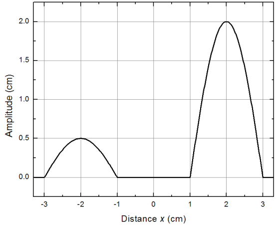




Lecture Question 16.2

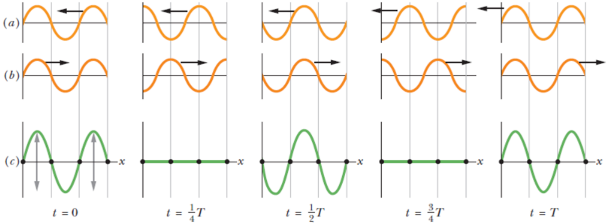
Two waves are traveling along a string. The left wave is traveling to the right at 0.5 cm/s and the right wave is traveling to the left at 2.0 cm/s. At what elapsed time will the two waves completely overlap and what will the maximum amplitude be at that time?

(a) 2.0 s, 1.5 cm
(b) 1.6 s, 2.5 cm
(c) 1.0 s, 1.5 cm
(d) 1.0 s, 2.5 cm
(e) 1.3 s, 0.0 cm



Standing Waves

When two waves travel on the same string in opposite directions, the result is a **standing wave**.



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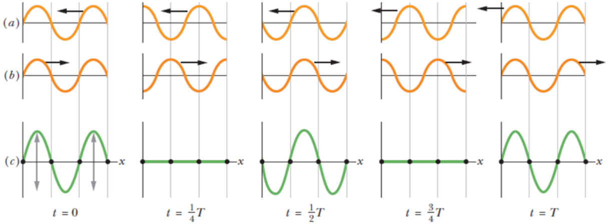

Nodes are spots where the displacement is always zero, and **anti-nodes** are the spots of maximum amplitude.

Standing Waves

Algebraically, a standing wave looks like:

$$\begin{aligned} y'(x,t) &= y_1(x,t) + y_2(x,t) \\ &= y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t) \end{aligned}$$

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The amplitude term gives the nodes and anti-nodes.

Standing Waves

A **node** *is when the amplitude is always zero:*

$$\begin{aligned} \sin(kx) &= 0 \\ kx &= n\pi \text{ for } n = 0, 1, 2, \ldots \\ x &= n\pi/k = n\lambda/2 \end{aligned}$$

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A **node** *is when the amplitude is always zero:*

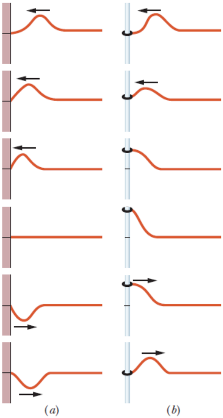
$$\begin{aligned} \sin(kx) &= 0 \\ kx &= n\pi \text{ for } n = 0, 1, 2, \ldots \\ x &= n\pi/k = n\lambda/2 \end{aligned}$$

An **anti-node** *is when the amplitude is maximum:*

$$\begin{aligned} \sin(kx) &= 1 \\ kx &= (n + 1/2)\pi \text{ for } n = 0, 1, 2, \ldots \\ x &= (n + 1/2)\pi/k = (n + 1/2)\lambda/2 \end{aligned}$$

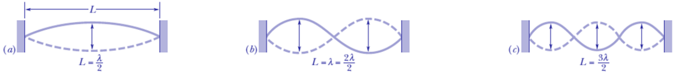
Standing Waves

When a rope reflects at a barrier, the result depends on the nature of the barrier.



Standing Waves

When a rope oscillates with two fixed points, certain frequencies called **harmonics** *result in standing waves with nodes and large anti-nodes.*



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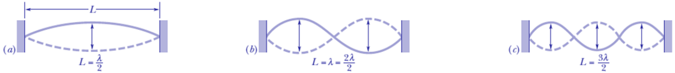

$$\lambda = \frac{2L}{n} \text{ for } n = 1, 2, 3 \ldots$$

$$f = \frac{v}{\lambda} = n\frac{v}{2L}$$

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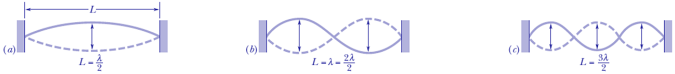

$$\lambda = \frac{2L}{n} \text{ for } n = 1, 2, 3 \ldots$$

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If we know the wave velocity (string tension and density), we can predict the harmonic frequencies.

Standing Waves