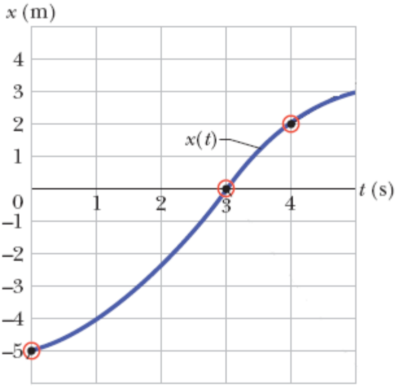
Chapter 3 - Motion Along a Straight Line

"I can calculate the motion of heavenly bodies, but not the madness of people."
- *Isaac Newton*

David J. Starling
Penn State Hazleton
PHYS 211

Position, Displacement and Distance

Kinematics *is the study of motion.*

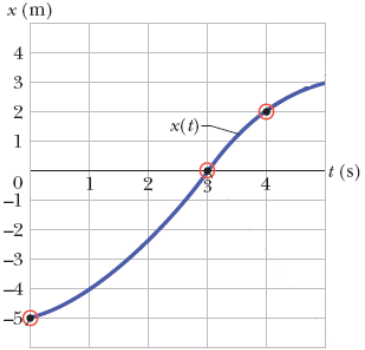


Position is the coordinate $x(t)$ of the object in question.

Position, Displacement and Distance

Displacement Δx *is the change in the position of an object.*

$$\Delta x = x_2 - x_1 \text{ (can be negative!)}$$



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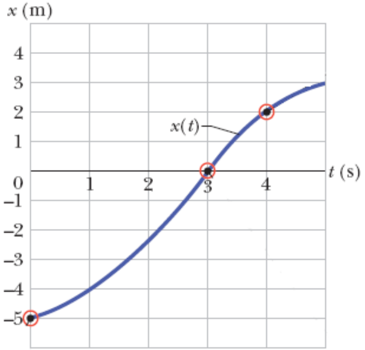
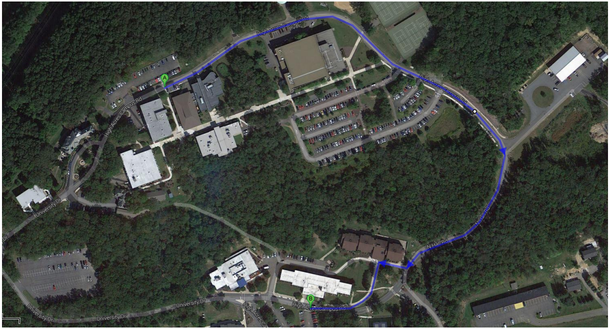


What is the displacement from time $t_1 = 0$ to $t_2 = 4$?

Position, Displacement and Distance

Distance *d is the total amount of ground an object covers during its motion.*



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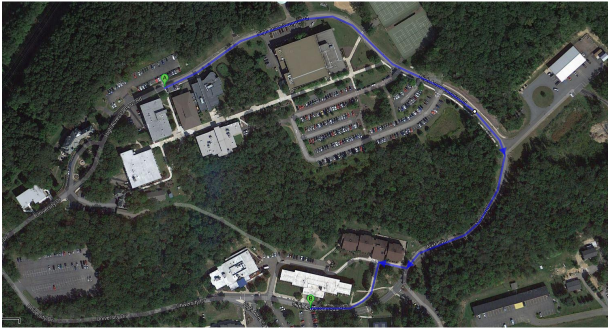


Is distance ever less than displacement?

Position, Displacement and Distance

Summary:

- Position is a function: $x(t)$
- Displacement is the change in position: $\Delta x = x_2 - x_1$
- Distance is how much ground is covered: d, always positive!

Position, Displacement and Distance

Lecture Question 3.1

A race car, traveling at constant speed, makes one lap around a circular track of radius r in a time t. Which one of the following statements concerning this car is true?

(a) The displacement is constant.

(b) The instantaneous velocity is constant.

(c) The average speed is the same over any time interval.

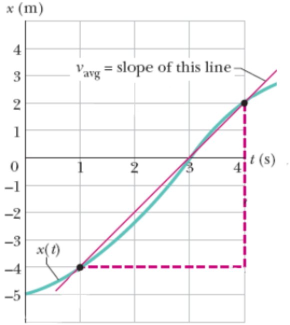
(d) The average velocity is the same over any time interval.

(e) The average speed and the average velocity are equal over the same time interval.

Average Velocity and Speed

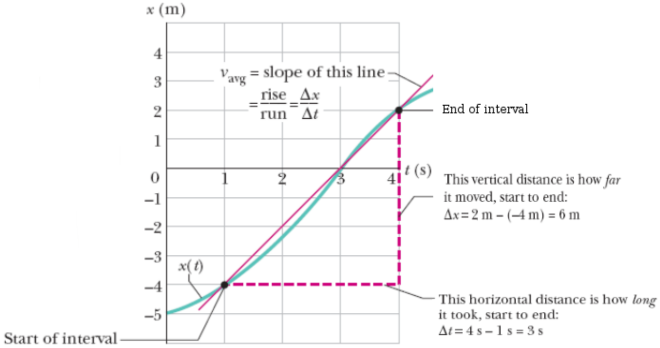
Average Velocity v_{avg} *is the displacement divided by the time interval.*

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$



Average Velocity and Speed

Find the average velocity:



Average Velocity and Speed

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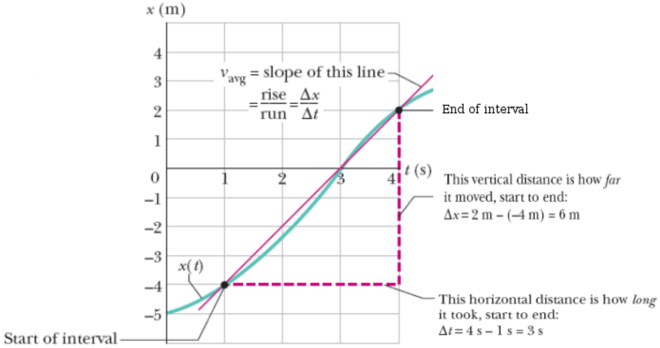

$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{2 - (-4)}{4 - 1} = 2 \text{ m/s}$$

Average Velocity and Speed

Average Speed s_{avg} *is the distance divided by the time interval.*

$$s_{avg} = \frac{d}{\Delta t} > 0$$

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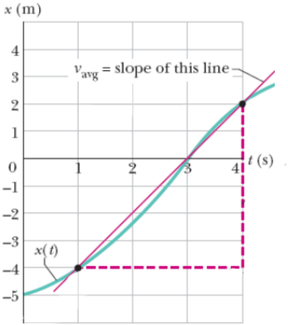

How does s_{avg} compare to v_{avg}?

Average Velocity and Speed

You drive for 8.4 km down a road at 70 km/h before you run out of gas. You walk another 2.0 km in 30 minutes. What is your overall displacement during this time?

(a) 2.0 km

(b) 2.1 km

(c) 10 km

(d) 590 km

Average Velocity and Speed

You drive for 8.4 km down a road at 70 km/h before you run out of gas. You walk another 2.0 km in 30 minutes. How long does this take?

(a) 0.12 hr

(b) 0.50 hr

(c) 30.12 min

(d) 0.62 hr

(e) 30.12 hr

Average Velocity and Speed

Lecture Question 3.2

You drive for 8.4 km down a road at 70 km/h before you run out of gas. You walk another 2.0 km in 30 minutes. What is your average velocity during this time?

(a) 4 km/hr

(b) 17 km/hr

(c) 37 km/hr

(d) 70 km/hr

Instantaneous Velocity and Speed

Instantaneous velocity *v is the average velocity during an infinitely short time period.*

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

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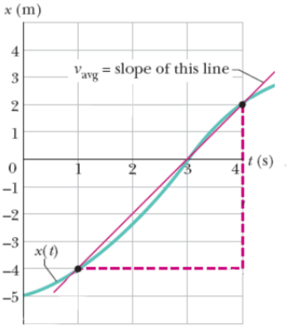
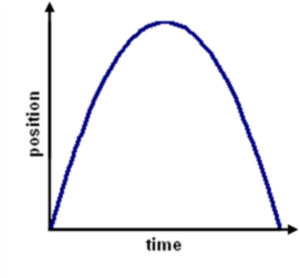


Instantaneous Velocity and Speed

Like average velocity, instantaneous velocity v has a "direction" and can be negative.



Instantaneous Velocity and Speed

Like average velocity, instantaneous velocity v has a "direction" and can be negative.

Instantaneous velocity is the slope of this curve at each moment in time!

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The **magnitude** of s is equal to the **magnitude** of v since $d = |\Delta x|$ during a short period of time.

$$s = v = \frac{dx}{dt}$$

Acceleration

Average acceleration a_{avg} *is the change in velocity divided by the time interval.*

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Colonel J. P. Stapp in a rocket sled.

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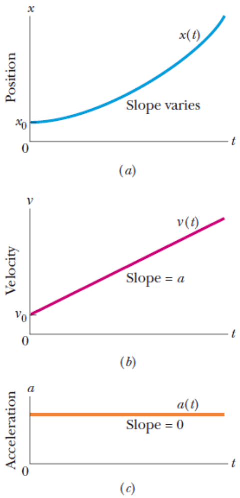

Acceleration

In many cases, $a =$ constant. In this case, we obtain three **very useful equations.**

$$a = a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} \rightarrow v_2 = v_1 + a\Delta t \quad (1)$$

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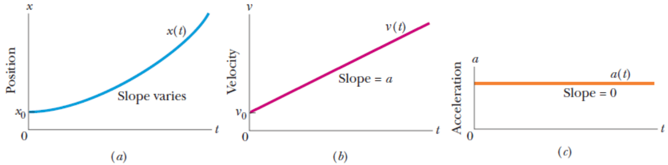
To replace v_{avg}, consider:



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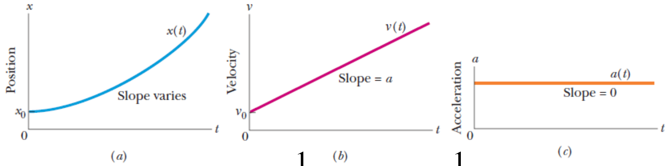

$$v_{avg} = v_1 + \frac{1}{2}\Delta v = v_1 + \frac{1}{2}a\Delta t$$

Acceleration

In many cases, $a =$ constant. In this case, we obtain three **very useful equations.**

$$x_2 = x_1 + v_1 \Delta t + \frac{1}{2} a \Delta t^2 \tag{2}$$

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Finally, if we eliminate Δt by combining Eqs. (1) and (2), we get:

$$\Delta t = \frac{v_2 - v_1}{a}$$

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$$\begin{aligned} \Delta t &= \frac{v_2 - v_1}{a} \\ x_2 &= x_1 + v_1 \left(\frac{v_2 - v_1}{a} \right) + \frac{1}{2} a \left(\frac{v_2 - v_1}{a} \right)^2 \end{aligned}$$

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$$v_2^2 = v_1^2 + 2a(x_2 - x_1) \tag{3}$$

Acceleration

The three **constant acceleration** *equations are:*

$$\begin{aligned} v_2 &= v_1 + at_2 \\ x_2 &= x_1 + v_1 t_2 + \frac{1}{2} a t_2^2 \\ v_2^2 &= v_1^2 + 2a(x_2 - x_1) \end{aligned}$$

where $t_1 = 0$ so that $\Delta t = t_2 - t_1 = t_2$.

Acceleration

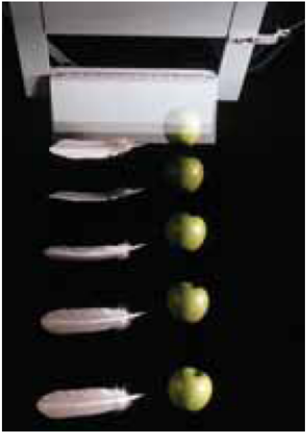
The three **constant acceleration** *equations are:*

$$
\begin{aligned}
v(t) &= v_0 + at \\
x(t) &= x_0 + v_0 t + \frac{1}{2}at^2 \\
v^2 &= v_0^2 + 2a(x - x_0)
\end{aligned}
$$

where $t_1 = 0$, $t_2 = t$, $x_1 = x_0$ and $x_2 = x$.

Acceleration

Objects near the surface of Earth accelerate toward the Earth with an acceleration of $g = 9.8$ *m/s*^{2} *(ignoring air resistance).*



Acceleration

Lecture Question 3.4

An explorer accidentally drops a wrench over the side of her hot air balloon as it rises from the ground. The balloon's upward acceleration is +4 m/s^2 with a a velocity of +2 m/s. Just after the wrench is released,

(a) its acceleration is -5.4 m/s^2, its velocity is +2 m/s.

(b) its acceleration is -5.4 m/s^2, its velocity is 0 m/s.

(c) its acceleration is -9.8 m/s^2, its velocity is +2 m/s.

(d) its acceleration is +5.4 m/s^2, its velocity is 0 m/s.

(e) its acceleration is 5.4 m/s^2, its velocity is -2 m/s.