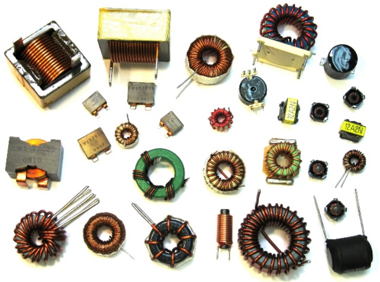
Induction and Inductance



The term **inductance** was coined by Oliver Heaviside in February 1886.

David J. Starling
Penn State Hazleton
PHYS 212

Faraday's Law of Induction

We have seen electric flux:

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

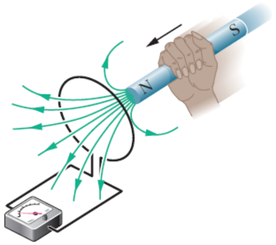
But we can define the magnetic flux in the same way:

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

- This is the flux through a *loop of wire*
- If $\vec{B}$ is uniform and perpendicular to the loop: $\Phi_B = BA$
- Magnetic flux has units of T m^2, also called the weber (Wb)

Faraday’s Law of Induction

What happens if I increase the flux through some loop?



- Current flows in the wire!
- The faster we change the flux, the bigger the current
- We induce an emf $\mathcal{E}$ in the loop:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Faraday's Law of Induction

What if there are N turns in my loop (solenoid)?

- Each turn has an induced emf, so we get:

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

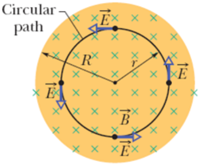
- We can change Φ_B by
  - Increasing/decreasing the magnetic field B
  - Increasing/decreasing the area A
  - Changing the tilt between $\vec{B}$ and $\vec{A}$

Faraday's Law of Induction

The induced emf in a loop is due to an electric field pushing charges around!

- Work done: $W = q\mathcal{E}$
- Also work done: $W = \int \vec{F} \cdot d\vec{s} = q \int \vec{E} \cdot d\vec{s}$
- Therefore: $\mathcal{E} = \int \vec{E} \cdot d\vec{s}$
- So Faraday's Law becomes:

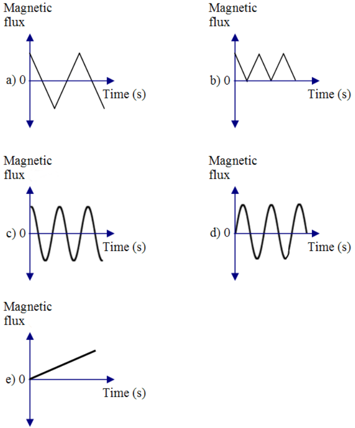
$$\mathcal{E} = \int \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

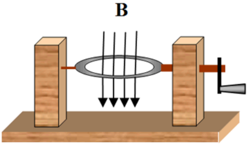


Faraday's Law of Induction

Lecture Question 13.1

The ring is rotated clockwise at a constant rate. Which graph best represents Φ_B?



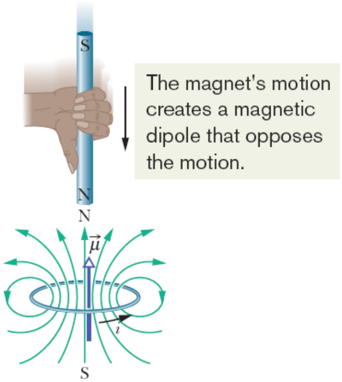


Lenz's Law

So what's with the negative sign?

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

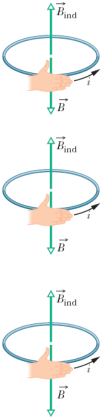
- The change in flux induces a current
- The induced current creates a magnetic field
- This induced magnetic field fights the change in flux
- There is an *opposition* to the change



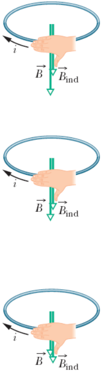
Lenz's Law

For an increasing $\vec{B}$ field

For a decreasing $\vec{B}$ field



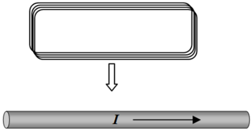
(a)



(b)

Lecture Question 13.2

A coil of wire approaches a long current at a constant speed.



(a) A current is induced in the coil.

(b) The rectangle will be pulled in the direction of the current in the wire.

(c) A magnetic force acts on the loop that pushes the loop into the page.

(d) There is no effect on the coil.

The Inductor

When we put a solenoid in a circuit, how does the circuit behave?

- If we pass a current through a solenoid, it produces a magnetic field, $B = \mu_0 n i$
- The flux through the solenoid is $\Phi_B = BA$
- If the solenoid has N loops, the "total flux" through the whole solenoid is $N\Phi_B$, called **magnetic flux linkage**
- The ratio of this total flux to the current is the inductance L:

$$L = \frac{N\Phi_B}{i}$$

- Compare to C:

$$C = \frac{q}{V}$$

The Inductor

For an ideal solenoid,

$$L = \frac{N\Phi_B}{i} = \frac{(nl)(BA)}{i} = \frac{nl(\mu_0 ni)A}{i} = \mu_0 n^2 lA$$

- The inductance has units of T-m^2/A, which we call a henry (H).
- Let's apply Faraday's Law to an inductor:

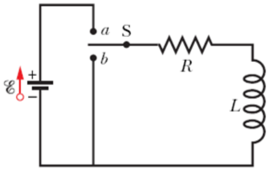
$$\mathcal{E} = -\frac{d(N\Phi_B)}{dt} = -\frac{d(Li)}{dt} = -L\frac{di}{dt}$$

- Apparently, an inductor produces its own emf
- This is called **self inductance**

The Inductor

In a circuit,

- an inductor acts similarly to a battery, producing an emf $\mathcal{E}(t) = -L\,di/dt$;
- we include it in the loop rule.



$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$
$$\frac{di}{dt} + \frac{R}{L}i = \frac{\mathcal{E}}{L}$$

- What is i?

The Inductor

Charging up the current:

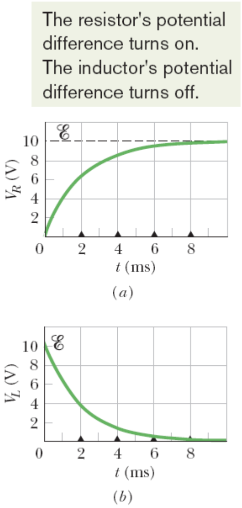
- The solution to

$$\frac{di}{dt} + \frac{R}{L}i = \frac{\mathcal{E}}{L}$$

is

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$$

- $\tau_L = L/R$.
- Compare to the RC circuit: $V = \mathcal{E}\left(1 - e^{-t/\tau}\right)$



The Inductor

Dis-charging the current:

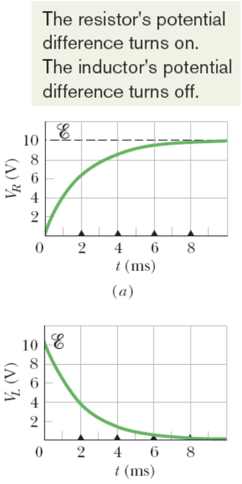
- The solution to

$$\frac{di}{dt} + \frac{R}{L} i = 0$$

is

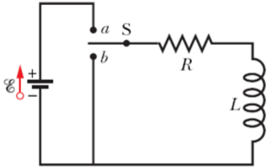
$$i(t) = \frac{\mathcal{E}}{R} e^{-t/\tau_L}$$

- $\tau_L = L/R$.
- Compare to the RC circuit:
  $V = \mathcal{E} e^{-t/\tau}$



(a)
(b)

The Inductor

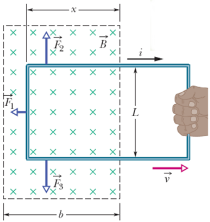


Key points:

- The inductor is slow to react
- Initially, the current is zero
- Eventually, it provides no resistance
- This process is exponential with $\tau = L/R$

Power and Mutual Induction

Consider a loop of wire being pushed at a speed v:



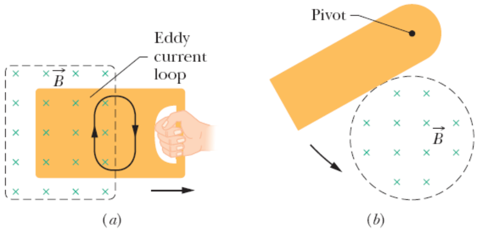
- An emf is induced in the loop

  $\mathcal{E} = d\Phi_B/dt = d(BLx)/dt = BLdx/dt = BLv$
- The resulting current is $i = \mathcal{E}/R = BLv/R$
- The resulting force is $F = iLB\sin(90°) = B^2L^2v/R$
- Power is just $P = Fv$, so

$$P = \frac{B^2L^2v^2}{R}$$

Power and Mutual Induction

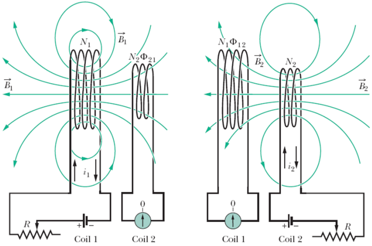
In fact, even if it's just a slab of metal, this emf still generates currents:



- These are called eddy-currents
- The eddy currents cause a force to oppose the motion

Power and Mutual Induction

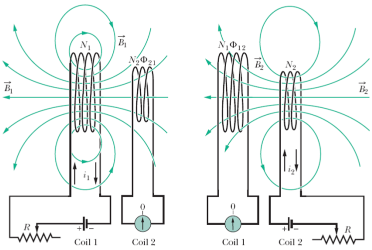
What if we put two coils next to each other?



- The magnetic field from coil 1 changes the flux through coil 2 Φ_{21}
- This induces a current in coil 2
- This is called **mutual inductance**:

$$M = \frac{N_2\Phi_{21}}{i_1} = \frac{N_1\Phi_{12}}{i_2}$$

Power and Mutual Induction



The mutual inductance

$$M = \frac{N_2 \Phi_{21}}{i_1} = \frac{N_1 \Phi_{12}}{i_2}$$

gives rise to an induced emf:

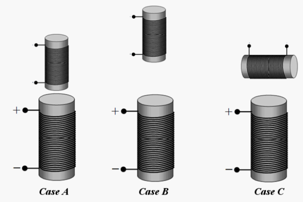
$$\mathcal{E}_2 = -M \frac{di_1}{dt}$$

$$\mathcal{E}_1 = -M \frac{di_2}{dt}$$

Power and Mutual Induction

Lecture Question 13.4

In each of the three cases shown, a time-varying current is flowing through the larger coil that produces a magnetic field. Which orientation has the largest mutual inductance?



(a) Case A

(b) Case B

(c) Case C

(d) All the same

(e) Not enough information