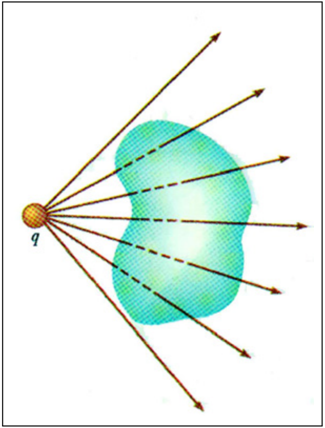
Gauss's Law



"I have had my results for a long time, but I do not yet know how I am to arrive at them."

- *Carl Friedrich Gauss*

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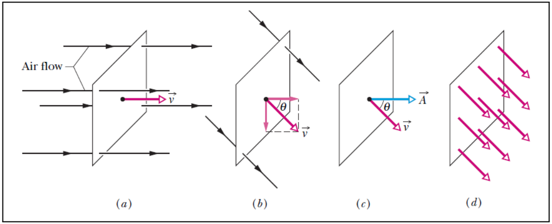
PHYS 212

Flux

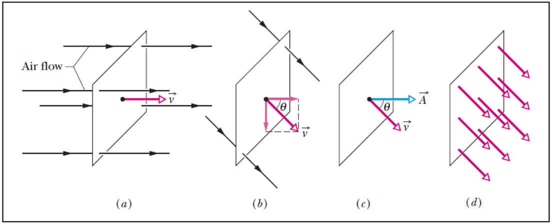
Before we jump right into Gauss's Law, we need to know about Flux.

Flux:

- Describes the flow ($\vec{v}$) of something through a surface ($\vec{A}$)
- The faster the flow, the larger the flux
- The bigger the surface, the larger the flux
- In general: $\Phi = \vec{v} \cdot \vec{A}$

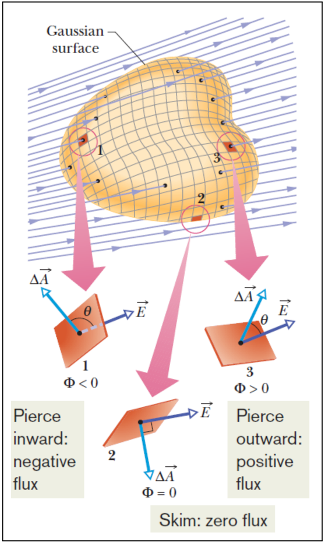


Flux



- Area vector $\vec{A}$ is perpendicular to the surface
- If $\vec{A}$ and $\vec{v}$ are parallel, Φ is maximum
- The bigger the angle between, the smaller the flux
- Think: sunlight on a solar panel

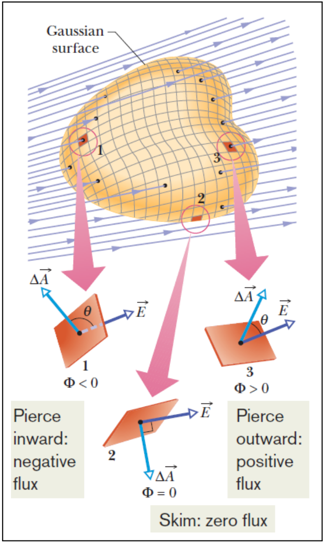
Flux



- Consider an arbitrary closed surface in space
- The electric field pierces this surface, just like water through a net
- The *flux* through the surface is the same:

$$\Phi = \vec{E} \cdot \vec{A}.$$

Flux



More generally:

$$\Phi = \sum \vec{E} \cdot \vec{A}$$

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

That is, we add up all of the products $\vec{E} \cdot \vec{A}$ for each small part of the surface.

If the parts become small, this becomes an integral!

Lecture Question 6.1

In the summer, Joe sets up his array of solar panels to maximize the amount of electricity output from the array when the Sun was high in the sky. In the winter, Joe finds that the array doesn't operate as well. What is the most likely cause of Joe's winter problem?

(a) Less sunlight reaches the Earth during the winter months.

(b) The sun is lower in the sky during the winter, so sunlight strikes the solar panels at an angle.

(c) The average temperature is much colder during the winter months.

(d) The Sun is not as bright during winter months as it is during the summer months.

Gauss's Law

Why do we care about flux?

- The flux through a closed surface is related to the charge within that surface!

Gauss's Law

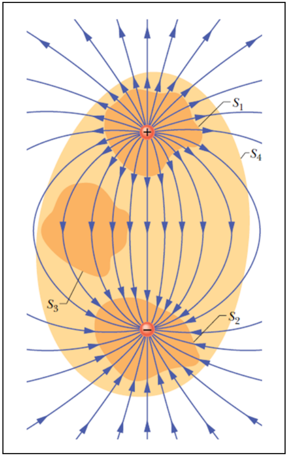
$$\Phi = q_{enc}/\epsilon_0$$

$$\oint \vec{E} \cdot d\vec{A} = q_{enc}/\epsilon_0$$

- $q_{enc} = q_1 + q_2 + q_3 + \ldots$
- $\Phi > 0$ implies total enclosed charge is positive
- $\Phi < 0$ implies total enclosed charge is negative

Gauss's Law

Two point charges:



What is the sign of the flux for each surface?

(a) $\Phi_1 > 0, \Phi_2 < 0,$
$\Phi_3 = 0, \Phi_4 > 0$

(b) $\Phi_1 > 0, \Phi_2 < 0,$
$\Phi_3 = 0, \Phi_4 < 0$

(c) $\Phi_1 > 0, \Phi_2 < 0,$
$\Phi_3 = \Phi_4 = 0$

(d) $\Phi_1 < 0, \Phi_2 > 0,$
$\Phi_3 = \Phi_4 = 0$

Gauss's Law implies Coulomb's Law?

Is Gauss's Law consistent with Coulomb's Law?

$$\epsilon_0 \Phi = q_{enc} \overset{?}{\rightarrow} E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \tag{1}$$

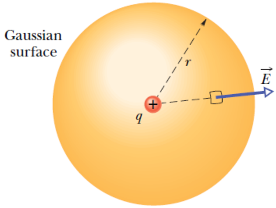


Fig. 23-8 A spherical Gaussian surface centered on a point charge q.

Gauss's Law implies Coulomb's Law?

We start with Gauss's Law, then make symmetry arguments:

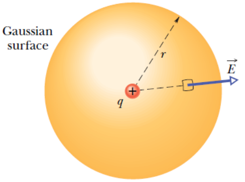


Fig. 23-8 A spherical Gaussian surface centered on a point charge q.

$$
\begin{aligned}
\epsilon_0 \Phi &= q_{enc} \\
\epsilon_0 \oint \vec{E} \cdot d\vec{A} &= q \\
\epsilon_0 \oint E dA &= q \\
\epsilon_0 E \oint dA &= q \\
\epsilon_0 E (4\pi r^2) &= q \\
E &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}
\end{aligned}
$$

Flux

Lecture Question 6.2

When calculating flux for Gauss's Law, which direction should the normal vector point?

(a) Parallel to the surface at each point.

(b) Perpendicular to the surface at each point, facing in.

(c) Perpendicular to the surface at each point, facing out.

(d) At an arbitrary angle to the surface at each point, but facing in.

(e) At an arbitrary angle to the surface at each point, but facing out.

Charge on a Conductor

What do we know about conductors?

- Conductors let charge move "freely"
- Inside a conductor, the electric field is generally zero. Why?
  - Otherwise, charge (the conduction electrons) would always be flowing in a conductor!
  - Initially, charge does flow when new charge is added
  - Eventually, this flow stops, and the charges are in **electrostatic equilibrium**

Charge on a Conductor

Let's look at a charged, isolated conductor:

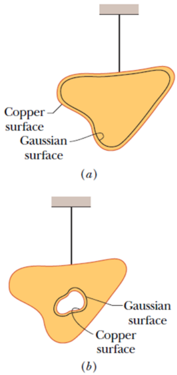


Fig. 23-9 (*a*) A lump of copper with a charge q hangs from an insulating thread. A Gaussian surface is placed within the metal, just inside the actual surface. (*b*) The lump of copper now has a cavity within it. A Gaussian surface lies within the metal, close to the cavity surface.

If the field is zero, isn't the *flux* zero?

Therefore, the enclosed charge is zero!

How does this change if there is a cavity?

Applications of Symmetry

For spherical symmetry, we have two **Shell Theorems**:

- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.
- If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.