Chapter 16 - Waves

"I'm surfing the giant life wave."
-William Shatner

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PHYS 213

Objectives (Ch 16)

Objectives for Chapter 16

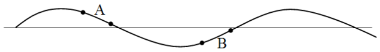
(a) Determine the wavelength and wave speed from time- and/or position-graphs of a wave or when provided similar information about the wave; in particular, relate frequency, wavelength and wave speed.

(b) Use the principle of linear superposition of waves to analyze the result of the interference of multiple waves.

(c) Determine the effect on wave speed and frequency of oscillation of a standing wave when changing one or more of these variables: length of the standing wave, mass/length of the medium, tension of the medium.

Objectives (Ch 16)

A wave is traveling along a rope to the right and is shown at a particular instant below. Two segments are labeled. Which of the following statements correctly describes the motion of the particles that compose the rope in these segments?



(a) Segment A: downward, segment B: upward.

(b) Segment A: upward, segment B: upward.

(c) Segment A: downward, segment B: downward.

(d) Segment A: upward, segment B: downward.

(e) Segment A: toward the left, segment B: toward the right.

The Basics of Waves

There are three main types of waves in physics:

(a) Mechanical waves: described by Newton's laws and propagate through matter, such as water, sound and seismic waves.

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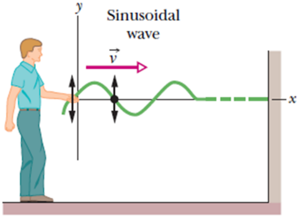
(c) Matter waves: described my quantum mechanics, these waves explain the wave nature of fundamental particles (electrons, protons, etc).

The Basics of Waves

Mechanical waves come in two flavors:



transverse

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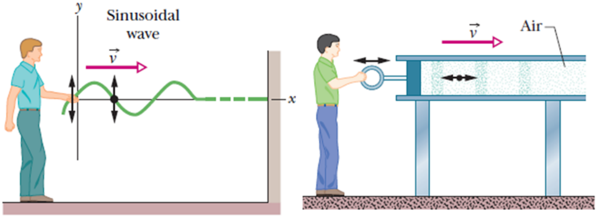

transverse **longitudinal**

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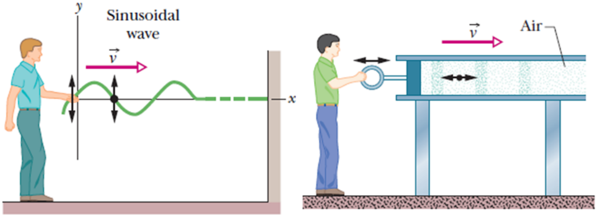

transverse **longitudinal**

The oscillations of matter are perpendicular or parrallel to the motion of the wave.

The Basics of Waves

All waves satisfy the so-called "wave equation."

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

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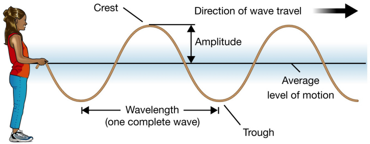
We are looking for $y(x, t)$.

The Basics of Waves

Mechanical waves are described by a traveling sine wave.

$$y(x,t) = y_m \sin(kx - \omega t)$$



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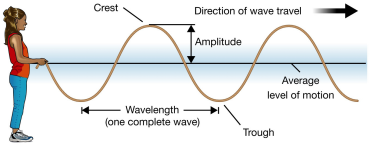
$$y(x,t) = y_m \sin(kx - \omega t)$$

- y_m: amplitude (maximum displacement)
- $k = 2\pi/\lambda$: wavenumber
- λ: wavelength
- $\omega = 2\pi f$: angular frequency



The Basics of Waves

Reminder: frequency, period and angular frequency are all related:

$$\begin{aligned} \omega &= 2\pi f \\ f &= \frac{1}{T} \\ \omega &= \frac{2\pi}{T} \end{aligned}$$

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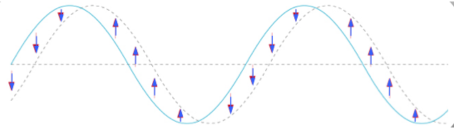
The wave number $k = 2\pi/\lambda$ is a "spatial frequency":

$$y(x, t = 0) = y_m \sin(kx + 0).$$

The Basics of Waves

Each part of the rope is confined to its x position and travels up and down like a harmonic oscillator:

$$y(x = 0, t) = y_m \sin(-\omega t)$$



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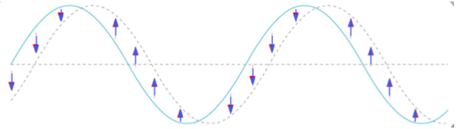
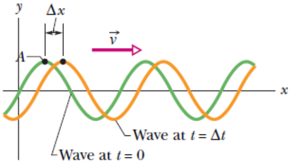


The speed of this part of the rope is just

$$u = \frac{dy(t)}{dt} = -\omega y_m \cos(\omega t)$$

The Basics of Waves

We can find the **speed of the wave** *from the argument of the sine function* $\sin(kx - \omega t)$, *known as the* **phase of the wave**.



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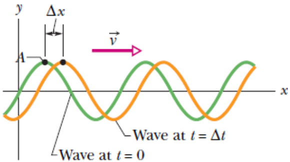

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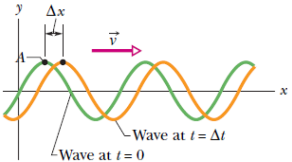


$$
\begin{aligned}
kx - \omega t &= constant \\
\frac{d}{dt}(kx - \omega t) &= 0 \\
k\frac{dx}{dt} - \omega &= 0 \\
\frac{dx}{dt} &= v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f
\end{aligned}
$$

The Basics of Waves

Alice and Bob are floating on a quiet river. At one point, they are 5.0 m apart when a speed boat passes. After the boat passes, they begin bobbing up and down at a frequency of 0.25 Hz. Just as Alice reaches her highest level, Bob is at his lowest level. As it happens, they are always within one wavelength. What is the speed of these waves?

(a) 1.3 m/s

(b) 2.5 m/s

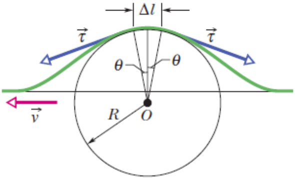
(c) 3.8 m/s

(d) 5.0 m/s

(e) 7.5 m/s

The Basics of Waves

We can derive the wave speed for a stretched rope with tension τ and mass density μ kg/m.



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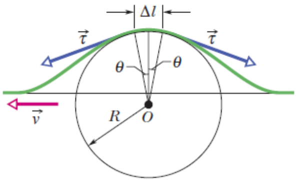

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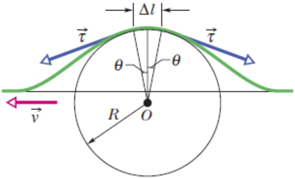

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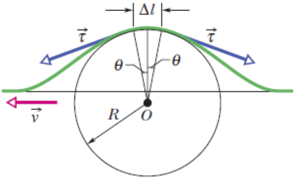

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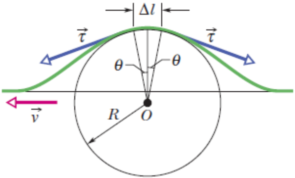

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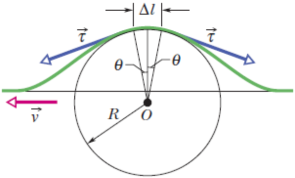

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The Basics of Waves

Example 1: A wave traveling along a string is described by

$$y(x,t) = 3.27\sin(72.1x - 2.72t)$$

with 3.27 in mm, 72.1 in rad/m and 2.72 in rad/s.

(a) What is the amplitude?

(b) What are the wavelength, period and frequency?

(c) What is the wave velocity?

(d) What is the displacement of the string at 22.5 cm and 18.9 s?

The Basics of Waves

Example 2: For the same wave,

$$y(x,t) = 3.27 \sin(72.1x - 2.72t),$$

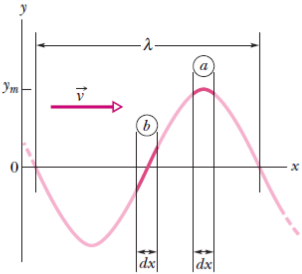
(a) what is the *transverse* velocity?

(b) what is the transverse acceleration?

Energy of Waves

Waves transfer energy in the direction of travel.

The rate of energy transfer is the power.



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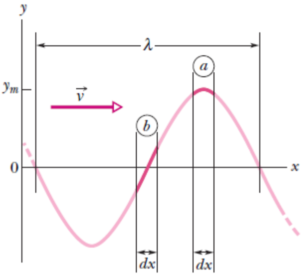

- The mass in region *b* has K but no U.
- The mass in region *a* has U but no K.

Energy of Waves

$$P_{avg} = \left(\frac{dK}{dt}\right)_{avg} + \left(\frac{dU}{dt}\right)_{avg}$$

Energy of Waves

$$\begin{aligned} P_{avg} &= \left(\frac{dK}{dt}\right)_{avg} + \left(\frac{dU}{dt}\right)_{avg} \\ &= 2\left(\frac{dK}{dt}\right)_{avg} \end{aligned}$$

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P_{avg} &= \frac{1}{2}\mu v \omega^2 y_m^2
\end{aligned}
$$

The wave generates power in proportion to its mass, velocity and the square of the frequency and amplitude.

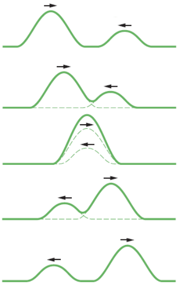
Energy of Waves

Example 3: A string of linear density 525 g/m is under a tension of 45 N. If a sinusoidal wave of 120 Hz with amplitude 8.5 mm is sent down its length, how much power does this wave transmit?

Interference of Waves

When two waves on a string overlap, their displacements add algebraically resulting in **interference***:*

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$



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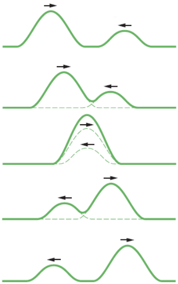

Note: the waves emerge without alteration.

Interference of Waves

Special case: two waves of equal magnitude, wavelength and velocity travel along the same string.

$$
\begin{aligned}
y'(x,t) &= y_1(x,t) + y_2(x,t) \\
&= y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi)
\end{aligned}
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&= y_m[\sin(kx - \omega t + \phi/2 - \phi/2) \\
&\quad + \sin(kx - \omega t + \phi/2 + \phi/2)]
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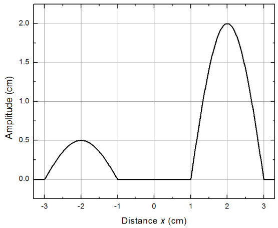
&= y_m[\sin(kx - \omega t + \phi/2 - \phi/2) \\
&\quad + \sin(kx - \omega t + \phi/2 + \phi/2)] \\
&= \underbrace{[2y_m \cos(\phi/2)]}_{\text{amplitude}} \underbrace{\sin(kx - \omega t + \phi/2)}_{\text{oscillation}}
\end{aligned}
$$

Interference of Waves

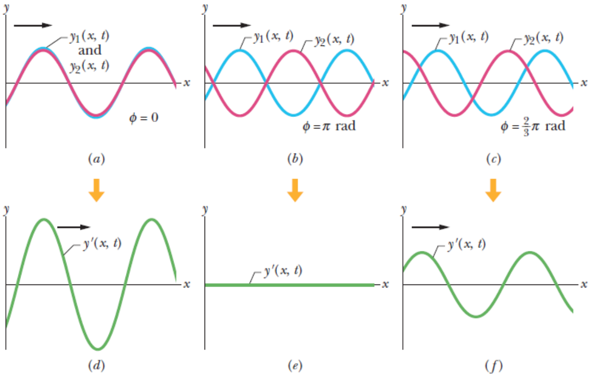
Two waves are traveling along a string. The left wave is traveling to the right at 0.5 cm/s and the right wave is traveling to the left at 2.0 cm/s. At what elapsed time will the two waves completely overlap and what will the maximum amplitude be at that time?

(a) 2.0 s, 1.5 cm
(b) 1.6 s, 2.5 cm
(c) 1.0 s, 1.5 cm
(d) 1.0 s, 2.5 cm
(e) 1.3 s, 0.0 cm



Interference of Waves

$$y'(x,t) = [2y_m \cos(\phi/2)] \sin(kx - \omega t + \phi/2)$$



Interference of Waves

Table 16-1

Phase Difference and Resulting Interference Types[a]

Phase Difference, in			Amplitude of Resultant Wave	Type of Interference
Degrees	Radians	Wavelengths		
0	0	0	$2y_m$	Fully constructive
120	$\frac{2}{3}\pi$	0.33	y_m	Intermediate
180	π	0.50	0	Fully destructive
240	$\frac{4}{3}\pi$	0.67	y_m	Intermediate
360	2π	1.00	$2y_m$	Fully constructive
865	15.1	2.40	$0.60y_m$	Intermediate

[a]The phase difference is between two otherwise identical waves, with amplitude y_m, moving in the same direction.

Interference of Waves

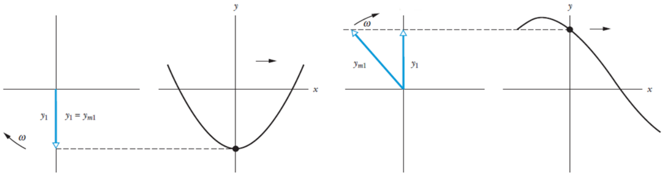
Example 4: Two identical sinusoidal waves travel along the same direction on a stretched rope. The amplitude of each wave is 9.8 mm and the phase difference between them is 100°.

(a) What is the amplitude of the resultant wave due to interference?

(b) What phase difference is required to have an amplitude of 4.9 mm?

Interference of Waves

We can represent a wave with a **phasor**, *a vector of length y_m that rotates about the origin at a frequency of ω.*



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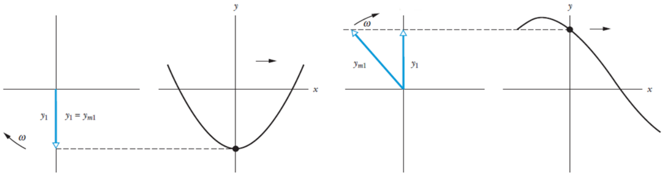
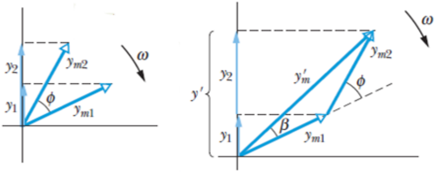


The vertical projection is the displacement of the wave at a particular point.

Interference of Waves

For two waves, you add the phasors to get the resulting displacement.



Interference of Waves

Example 5: Two waves $y_1(x,t)$ and $y_2(x,t)$ have the same wavelength and travel in the same direction along a string. Their amplitudes are $y_{m1} = 4.0$ mm and $y_{m2} = 3.0$ mm and their phase difference is $\pi/3$.

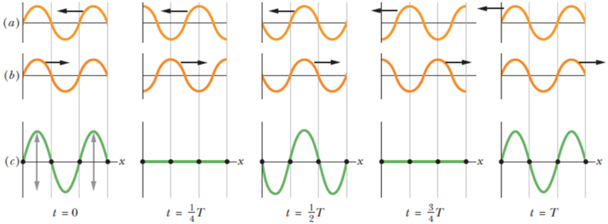
(a) Draw the phasor diagram for these two waves.

(b) Find the resulting amplitude and the phase constant.

(c) Write out the equation of the wave.

Standing Waves

When two waves travel on the same string in opposite directions, the result is a **standing wave**.



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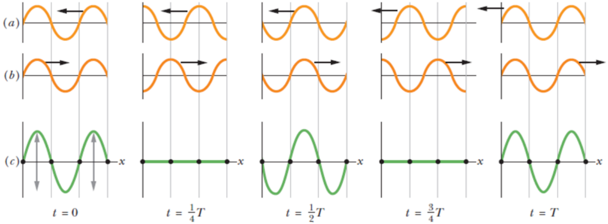

Nodes are spots where the displacement is always zero, and **anti-nodes** are the spots of maximum amplitude.

Standing Waves

Algebraically, a standing wave looks like:

$$\begin{aligned} y'(x,t) &= y_1(x,t) + y_2(x,t) \\ &= y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t) \end{aligned}$$

Standing Waves

Algebraically, a standing wave looks like:

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&= y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t) \\
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$$

The amplitude term gives the nodes and anti-nodes.

Standing Waves

A **node** *is when the amplitude is always zero:*

$$
\begin{aligned}
\sin(kx) &= 0 \\
kx &= n\pi \text{ for } n = 0, 1, 2, \ldots \\
x &= n\pi/k = n\lambda/2
\end{aligned}
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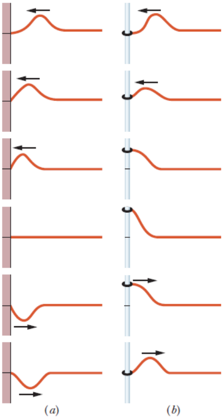
An **anti-node** *is when the amplitude is maximum:*

$$\begin{aligned} \sin(kx) &= 1 \\ kx &= (n + 1/2)\pi \text{ for } n = 0, 1, 2, \ldots \\ x &= (n + 1/2)\pi/k = (n + 1/2)\lambda/2 \end{aligned}$$

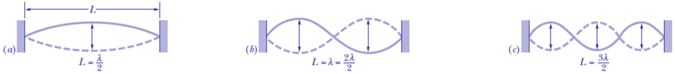
Standing Waves

When a rope reflects at a barrier, the result depends on the nature of the barrier.



Standing Waves

When a rope oscillates with two fixed points, certain frequencies called **harmonics** *result in standing waves with nodes and large anti-nodes.*



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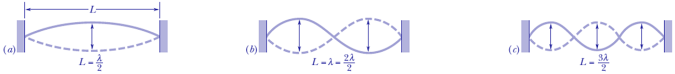

$$\lambda = \frac{2L}{n} \text{ for } n = 1, 2, 3 \ldots$$

$$f = \frac{v}{\lambda} = n\frac{v}{2L}$$

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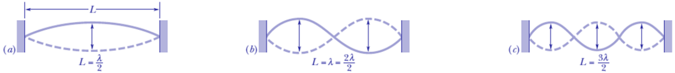

$$\lambda = \frac{2L}{n} \text{ for } n = 1, 2, 3 \ldots$$

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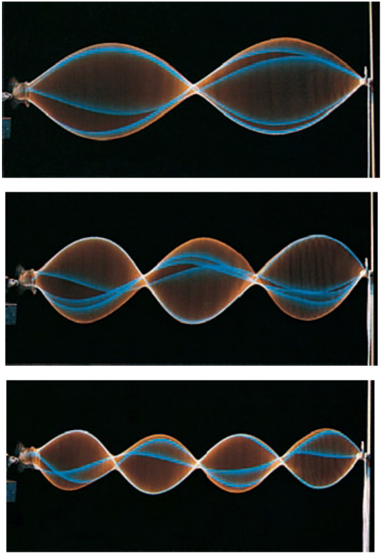
If we know the wave velocity (string tension and density), we can predict the harmonic frequencies.

Standing Waves



Standing Waves

Example 6: In the figure below, the string has mass 2.5 g and length 0.80 m. If the tension force is 325 N,

(a) What is the wavelength of the standing wave?

(b) Which harmonic n is this?

(c) What is the frequency of the wave?

(d) What is the maximum transverse velocity at $x = 0.180$ m?

(e) At what time is this velocity maximum?