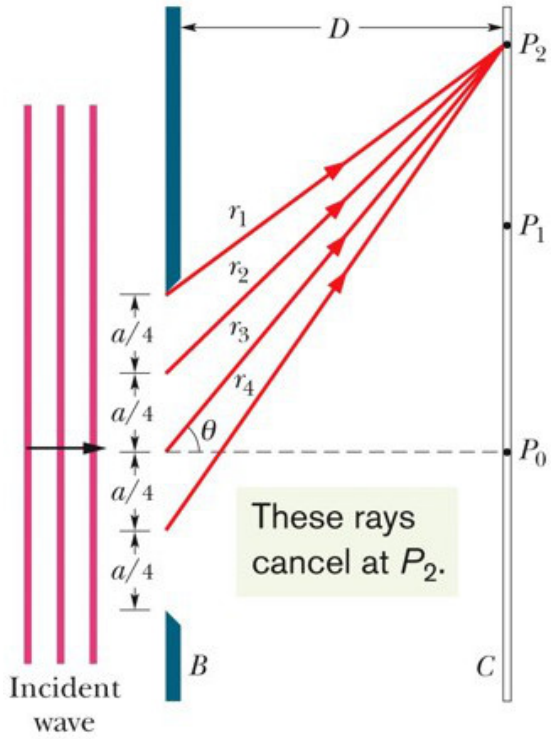
Chapter 4 - Diffraction

Diffraction is the phenomenon that occurs when a wave interacts with an obstacle.

David J. Starling
Penn State Hazleton
PHYS 214

Single Slit

When a wave interacts with an obstacle, the waves spread out and interfere.



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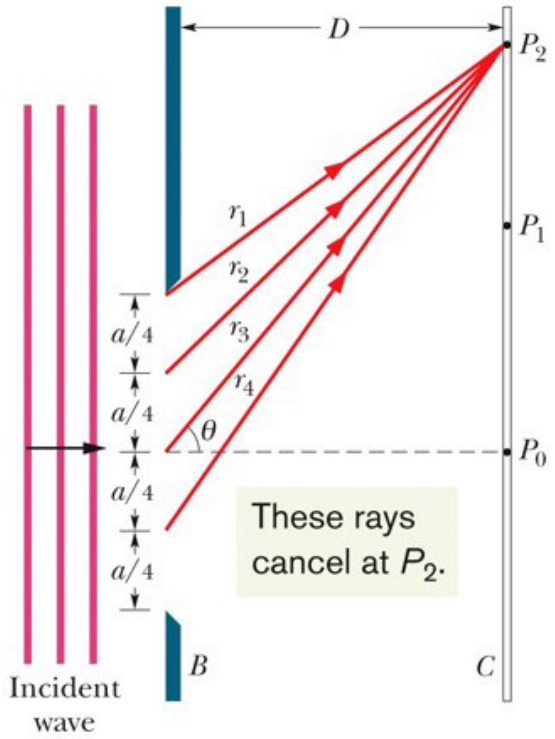
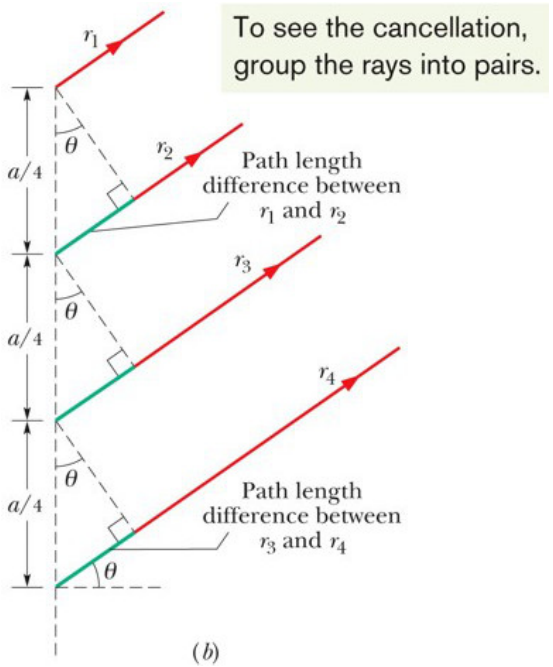


The obstacle should be of the same size as the wavelength in order to be observed.

Single Slit

Destructive interference occurs as specific points; these points are found geometrically.



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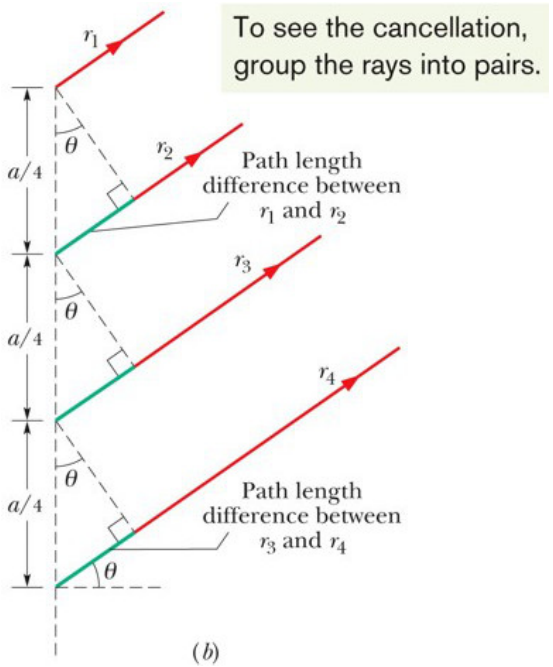


Careful consideration results in an equation for the minima:

$$a\sin(\theta) = m\lambda \text{ for } m = 1, 2, 3, \ldots \qquad (1)$$

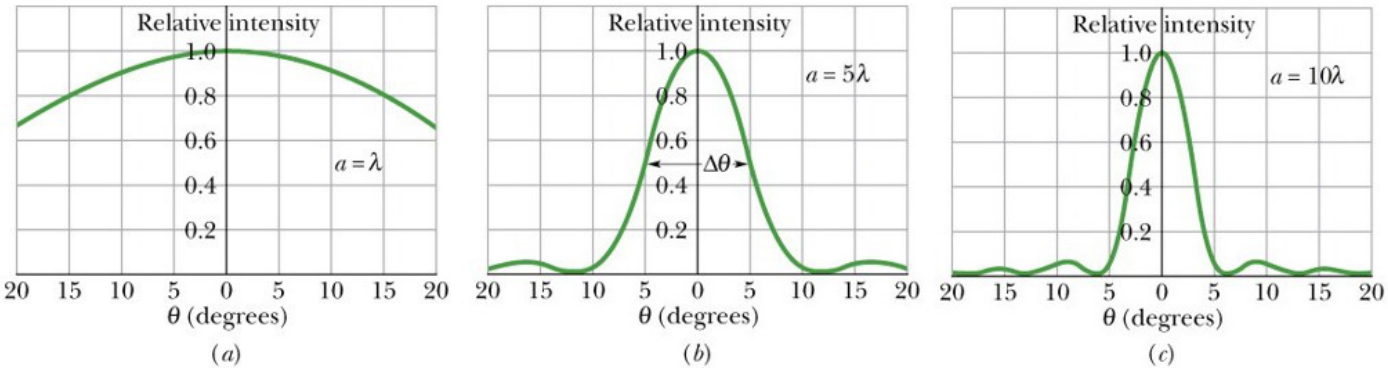
Single Slit

More generally, we can write the intensity at different angles:

$$I(\theta) = I_m \text{sinc}^2 \left[\frac{\pi a \sin(\theta)}{\lambda} \right] \qquad (2)$$

where

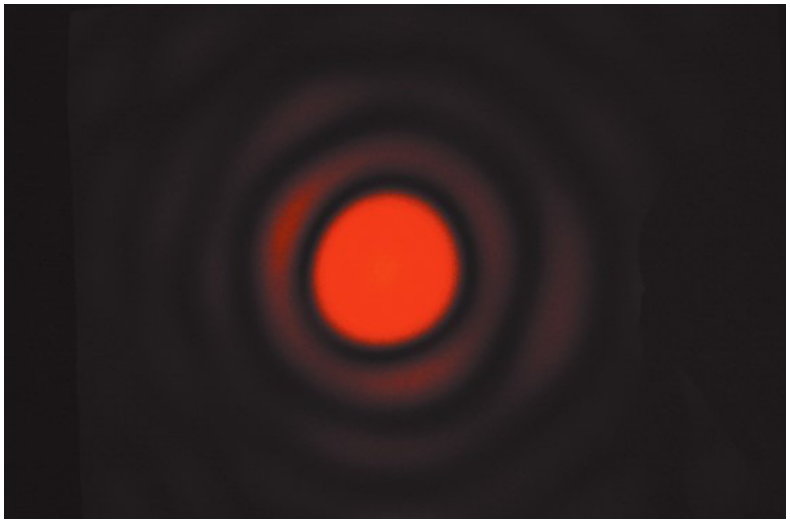
$$\text{sinc}(x) = \sin(x)/x.$$



Single Slit

For a circular aperture, the first minimum is

$$\sin(\theta) = 1.22\frac{\lambda}{a}. \tag{3}$$



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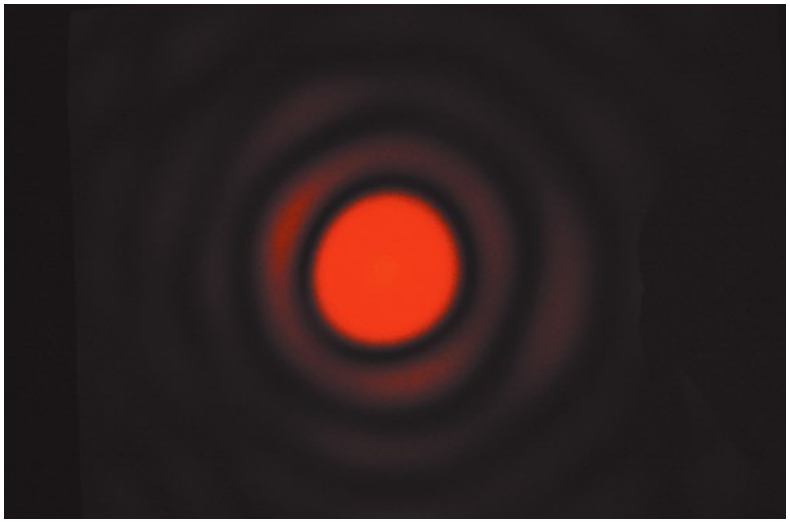
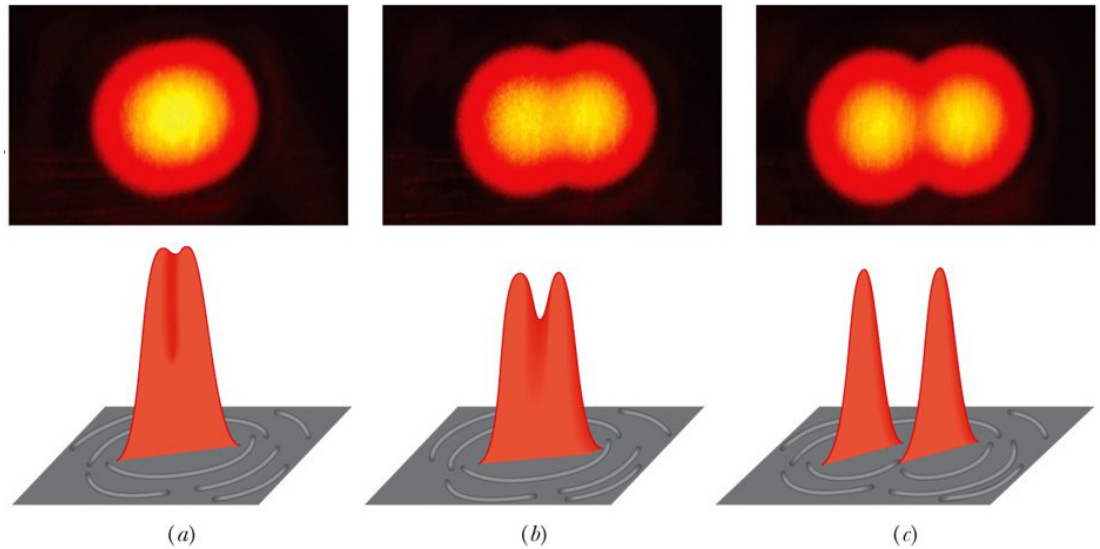


The 1.22 comes from the geometry of the interference. The full intensity profile is given by the Airy pattern.

Single Slit

For two objects separated by a small angle (think binary star system), the Rayleigh criterion says they can be distinguished only if:

$$\theta_R \geq 1.22\frac{\lambda}{d}. \qquad (4)$$



(a) (b) (c)

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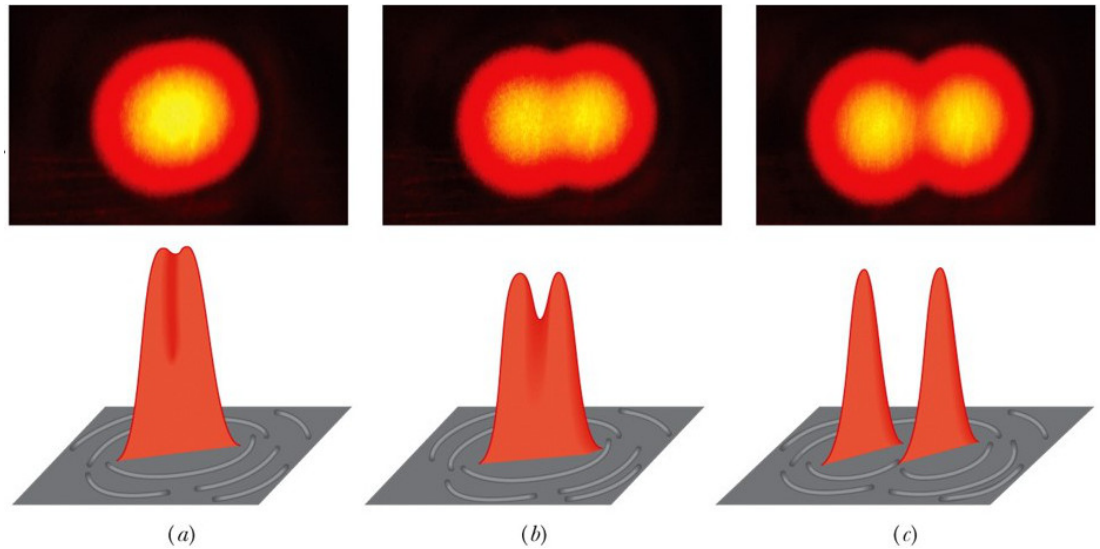
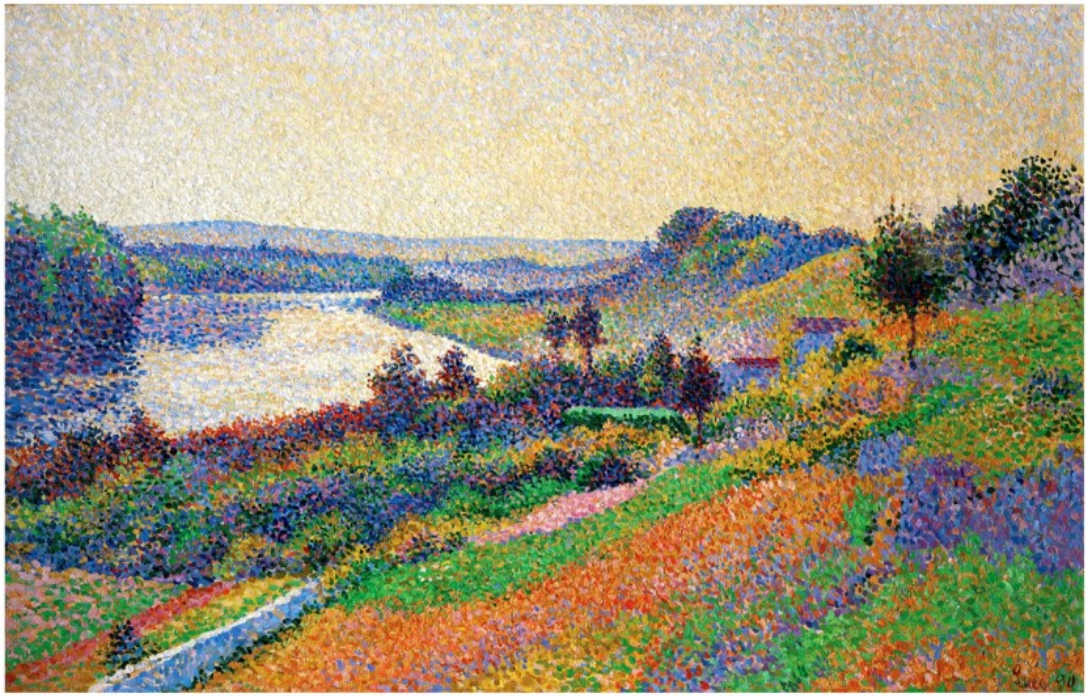


Here, d is the diameter of the optics used in the measurement.

Single Slit

In Art, we see how this comes into play with pointillism.



Maximilien Luce, *The Seine at Herblay*, 1890. Musée d'Orsay, Paris, France. Photo by Erich Lessing/Art Resource

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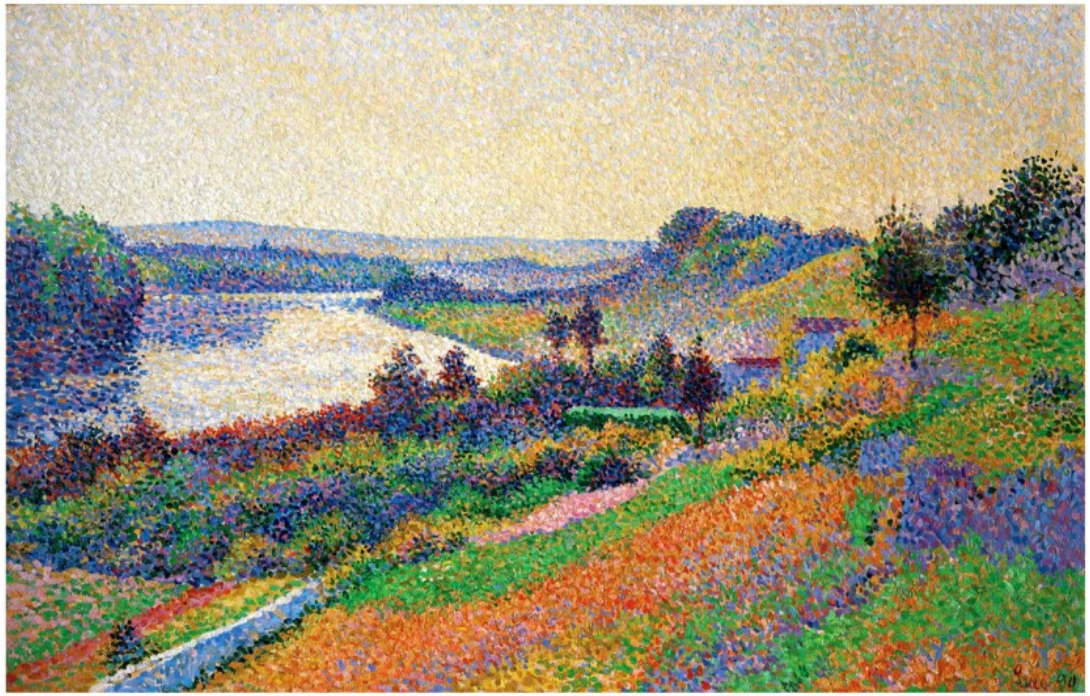
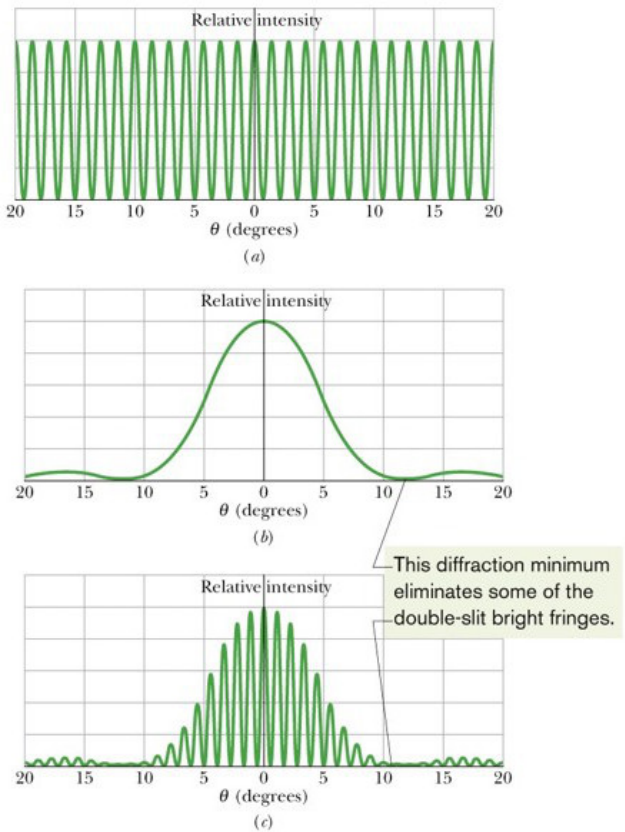


Maximilien Luce, *The Seine at Herblay*, 1890. Musée d'Orsay, Paris, France. Photo by Erich Lessing/Art Resource

Your pupil acts as an aperture, limiting your angular resolution.

Double Slit

When two slits are used, there is a combination of interference and diffraction.



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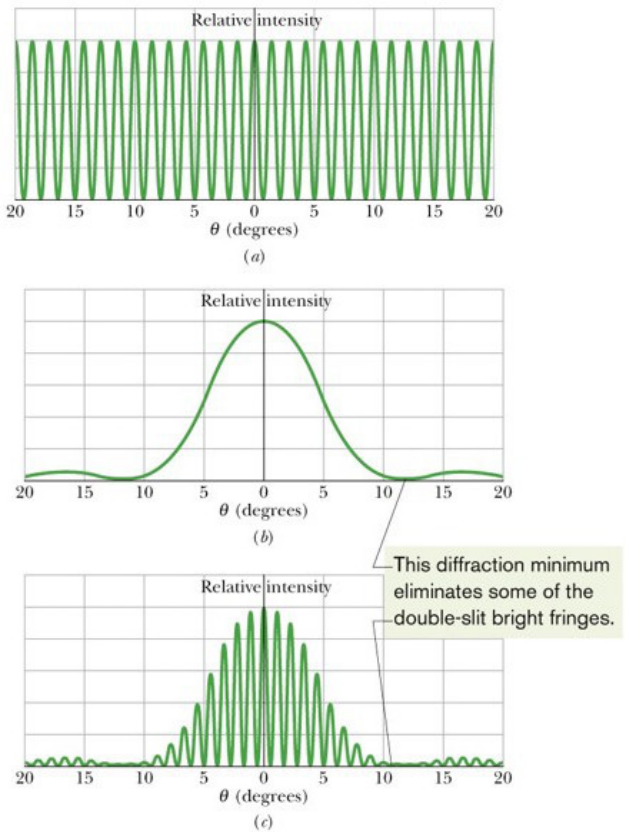
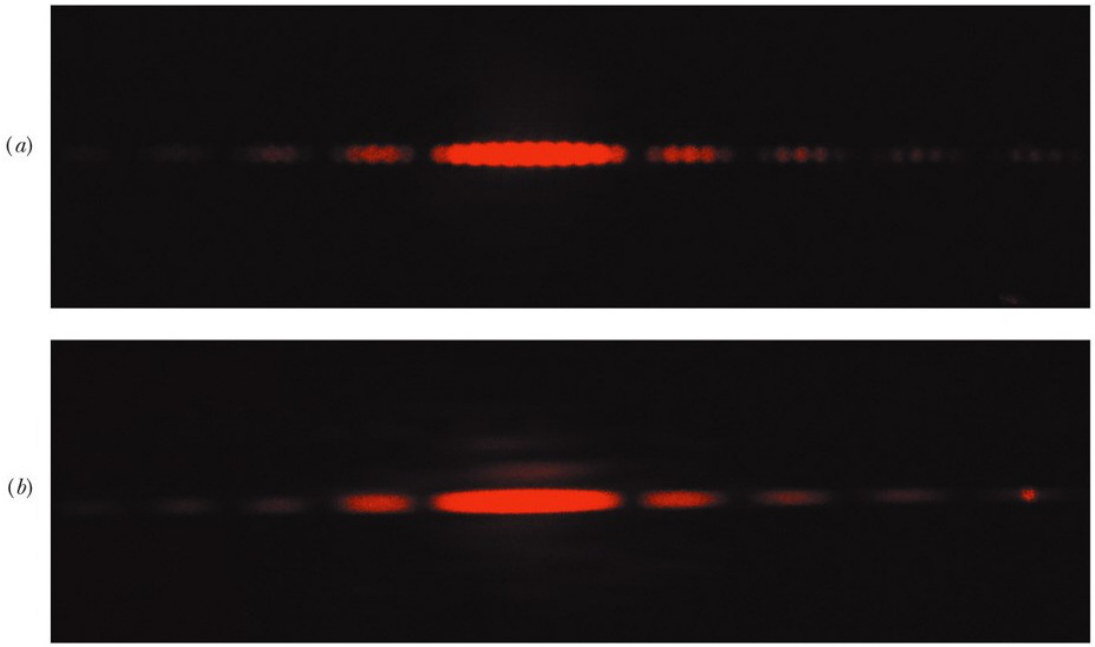


The more narrow the slits, the wider the diffraction pattern. For infinitely narrow slits, there is no diffraction.

Double Slit

Mathematically, we simply multiply the interference and the diffraction terms:

$$I(\theta) = I_m \underbrace{\cos^2\left[\frac{\pi d \sin(\theta)}{\lambda}\right]}_{\text{interference}} \underbrace{\text{sinc}^2\left[\frac{\pi a \sin(\theta)}{\lambda}\right]}_{\text{diffraction}} \qquad (5)$$



Double Slit

Lecture Question 4.1

In a single slit experiment, what effect on the diffraction pattern would result as the slit width is decreased?

(a) The width of the central band would increase.

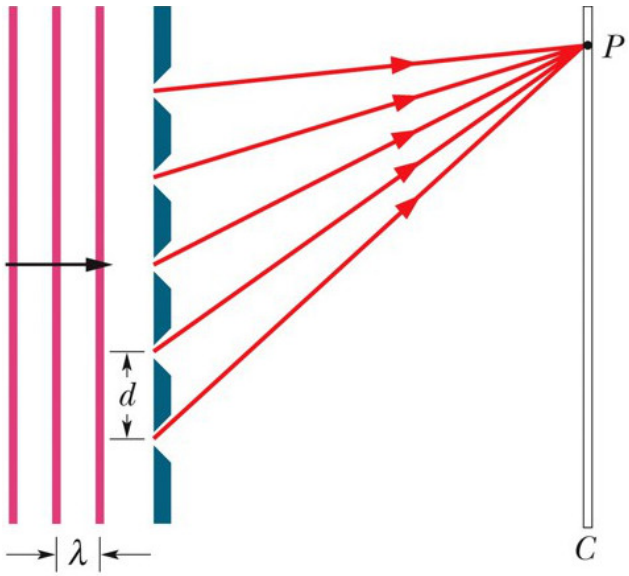
(b) The width of the central band would decrease.

(c) The width of the central band would not change.

(d) The result depends on the wavelength, so we cannot say.

Gratings

When many slits are placed together, this is known as a grating.



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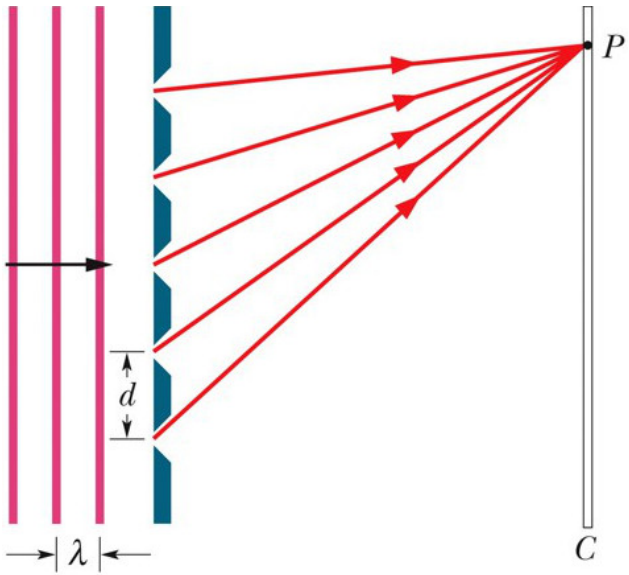


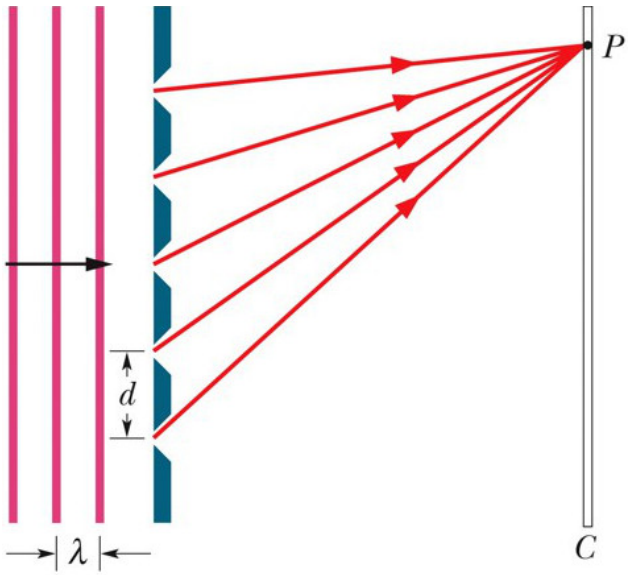
The maxima are located at an angle θ governed by the grating equation:

$$d\sin(\theta) = m\lambda, \text{ where } m = 0, 1, 2, \ldots \qquad (6)$$

Gratings

For a single wavelength, the half-width of the line is given by:

$$\Delta\theta_{hw} = \frac{\lambda}{Nd\cos(\theta)} \qquad (7)$$



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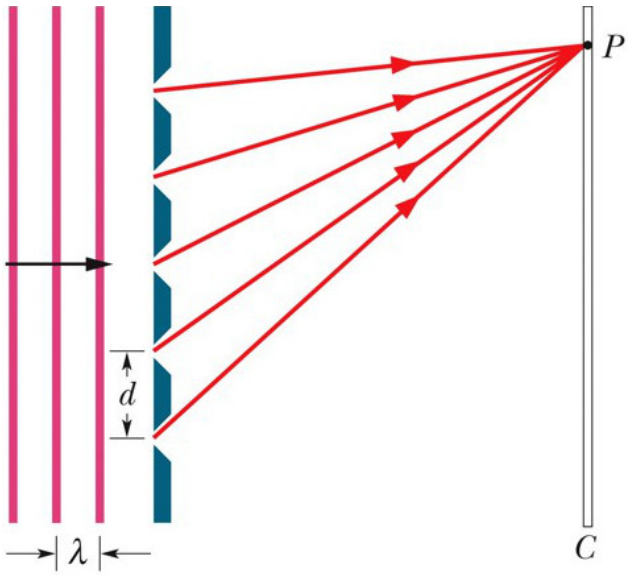

N is the number of lines on the grating.

Gratings

Gratings are mainly used to separate colors (like prisms).

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The dispersion D measures how well a grating separates each wavelength:

$$D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos(\theta)} \tag{8}$$

Gratings

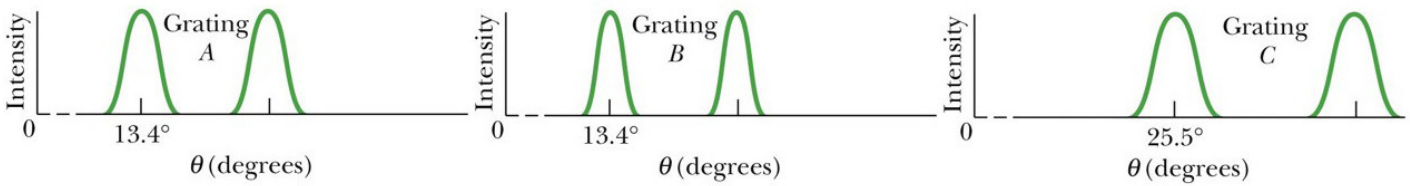
Another metric used for gratings is its ability to distinguish two close wavelengths, known as the resolving power.

$$R = \frac{\lambda_{avg}}{\Delta\lambda} = Nm \qquad (9)$$

Table 36-1 Three Gratings[a]

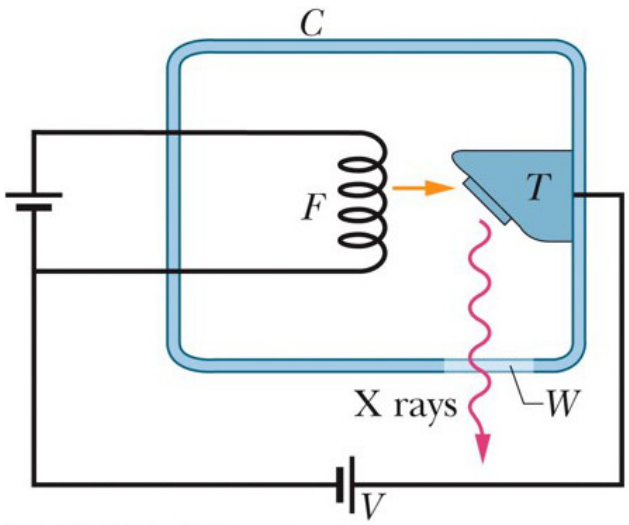
Grating	N	d (nm)	θ	D (°/μm)	R
A	10 000	2540	13.4°	23.2	10 000
B	20 000	2540	13.4°	23.2	20 000
C	10 000	1360	25.5°	46.3	10 000

[a]Data are for $\lambda = 589$ nm and $m = 1$.



X-Rays

X-Rays are high energy electromagnetic waves. They can be produced when electrons strike a metal surface.



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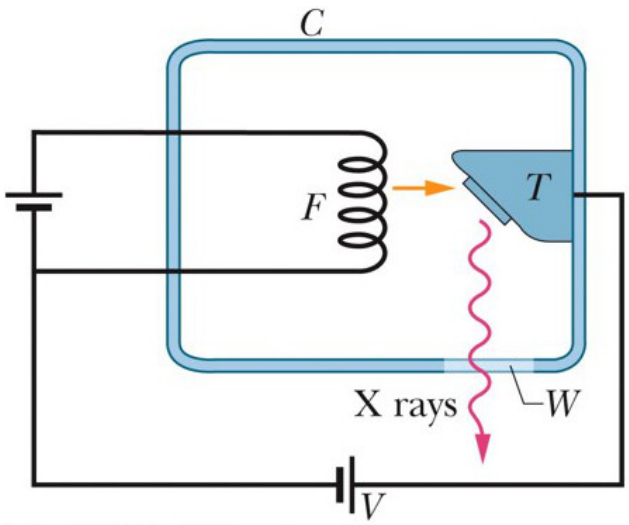
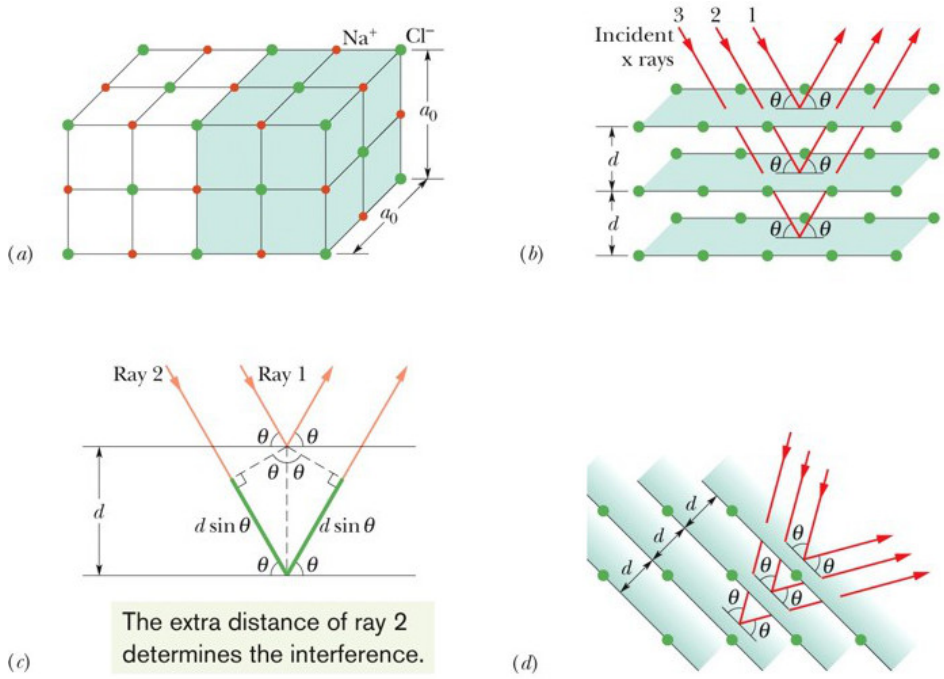


The wavelength of an x-ray is about 0.1 nm, or 1 Å.

X-Rays

When x-rays travel through a crystalline structure, they diffract off of the periodic layers.



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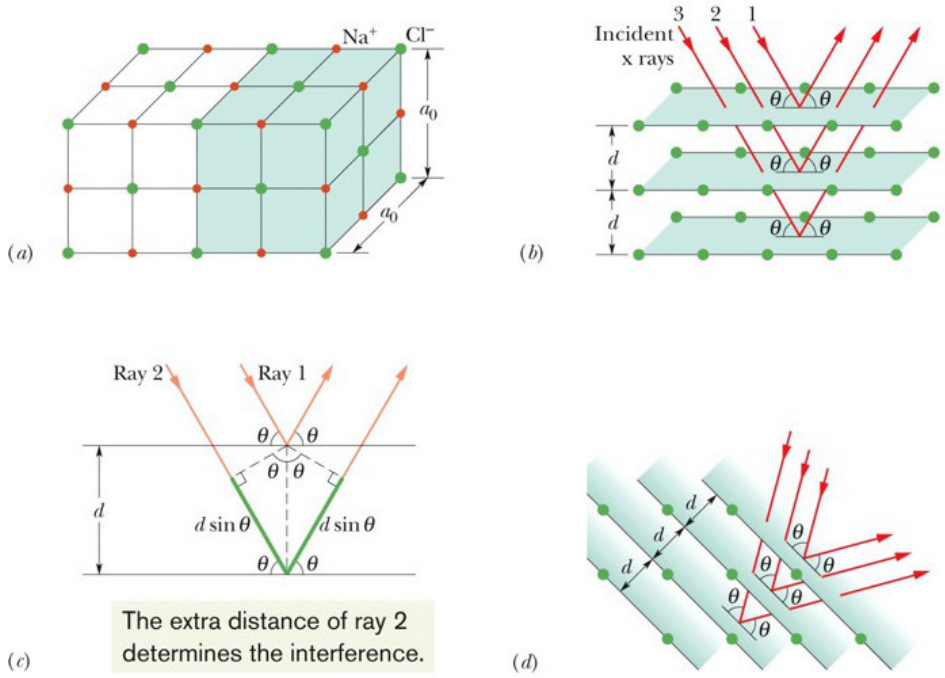

The excess distance determines the interference of the reflections of different layers:

$$2d\sin(\theta) = m\lambda \text{ where } m = 1, 2, 3, \ldots \qquad (10)$$

Double Slit

Lecture Question 4.2

Red and blue monochromatic beams of light are combined and then directed onto a diffraction grating. The pattern is observed on a screen located a behind the grating.

(a) The central maximum is purple. The maxima on each side would alternate, first red, then blue.

(b) The central maximum is purple. The maxima on each side would alternate, first blue, then red.

(c) The central maximum is red. The maxima on each side would alternate, first blue, then red.

(d) The central maximum is blue. The maxima on each side would alternate, first red, then blue.

(e) The central maximum is blue. The maxima on each side would alternate, first blue, then red.