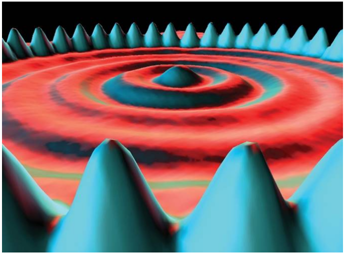
Chapter 8 - Atomic Structure



"Oh, the humanity!"

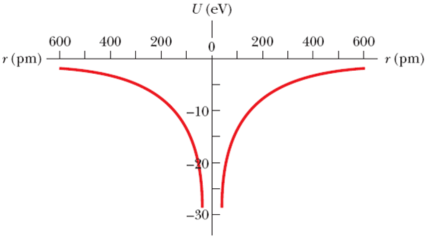
-Herbert Morrison, radio reporter of the Hindenburg disaster

David J. Starling
Penn State Hazleton
PHYS 214

Hydrogen Atom

The hydrogen atom is composed of a proton and an electron with potential energy:

$$U(r) = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}.$$



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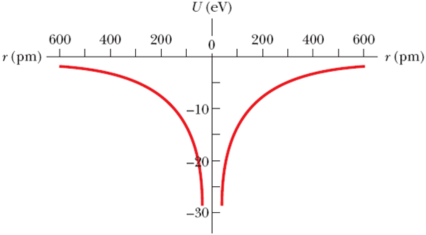

Now, our well is not square but follows an inverse law.

Hydrogen Atom

All we need to do is solve Schrödinger's equation in spherical coordinates with this radial potential.

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$$\frac{-\hbar^2}{2\mu}\frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial\phi^2}\right]-\frac{e^2}{4\pi\epsilon_0 r}\psi=E\psi$$

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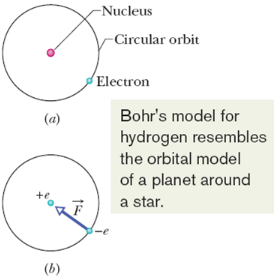
We're not there yet, so let's try a different way...

Hydrogen Atom

Instead, let's think of the electron as orbiting the proton classically.



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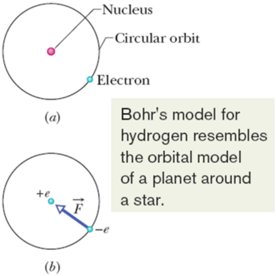


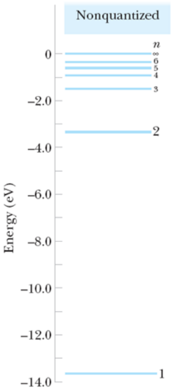
Using centripetal acceleration, we get

$$F = ma$$

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = m\left(-\frac{v^2}{r}\right)$$

Hydrogen Atom

Based on experimental evidence, he know that the energy levels are quantized.



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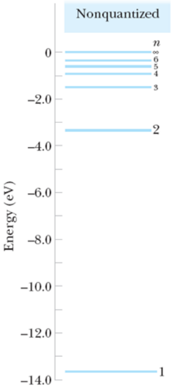

So let's guess that the angular momentum l is *also* quantized:

$$l = rmv = n\hbar$$
$$v = \frac{n\hbar}{rm}$$

for $n = 1, 2, 3, \ldots.$

Hydrogen Atom

Combining our quantized angular momentum assumption with the orbital equation, we get

$$r = \frac{4\pi\epsilon_0\hbar^2}{me^2}n^2 = \frac{\epsilon_0 h^2}{\pi me^2}n^2$$

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We call the multiplier of n^2 the Bohr Radius a.

$$r = an^2 \text{ with } a = \frac{\epsilon_0 h^2}{\pi me^2}.$$

and $a \approx 52.92$ pm.

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This result is surprisingly accurate!

Hydrogen Atom

To find the energy, we combine kinetic and potential and then use the orbital equation.

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = m\frac{v^2}{r}$$

$$E = \frac{1}{2}mv^2 - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r^2}$$

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Subbing in the Bohr radius formula for r, we get:

$$E_n = -\frac{me^4}{8\epsilon_0 h^2}\frac{1}{n^2} = -\frac{E_0}{n^2} \text{ for } n = 1, 2, 3, \ldots$$

where $E_0 = 2.180 \times 10^{-18}$J.

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This result agrees with quantum theory!

Hydrogen Atom

The hydrogen atom can absorb a photon if its energy matches an electron transition energy.

$$\begin{aligned} E_\gamma = hf &= E_{high} - E_{low} \\ \frac{hc}{\lambda} &= -E_0 \left(\frac{1}{n_{high}^2} - \frac{1}{n_{low}^2} \right) \\ \frac{1}{\lambda} &= -\frac{E_0}{hc} \left(\frac{1}{n_{high}^2} - \frac{1}{n_{low}^2} \right) \\ \frac{1}{\lambda} &= R \left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2} \right) \end{aligned}$$

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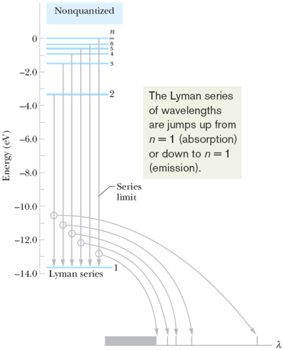
This is how we find the wavelength of photons emitted from electronic transitions.

Hydrogen Atom

Each transition has a unique energy with a photon of a different wavelength.



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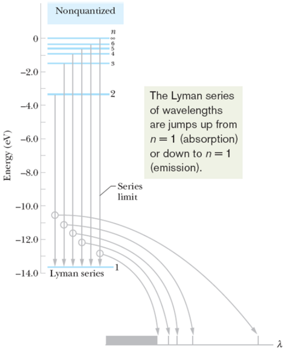
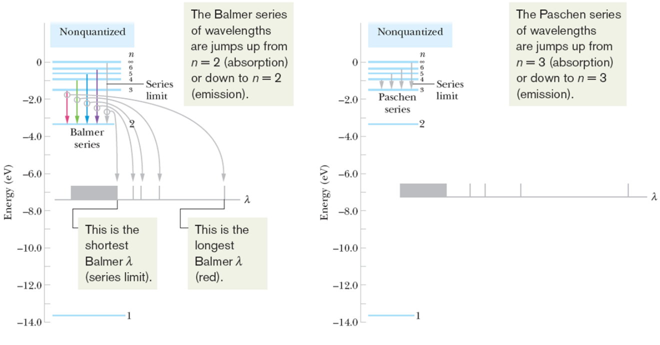


The Lyman series of wavelengths are jumps up from $n = 1$ (absorption) or down to $n = 1$ (emission).

If the ending state is $n = 1$, then that series of wavelengths is known as the Lyman series.

Hydrogen Atom

But the electron may end up in another state instead.



(c) (d)

Hydrogen Atom

Remember this?

$$\frac{-\hbar^2}{2\mu}\frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial\phi^2}\right]-\frac{e^2}{4\pi\epsilon_0 r}\psi=E\psi$$

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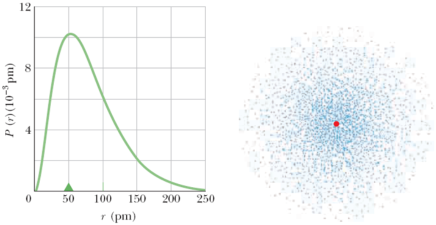
The *ground state* solution is:

$$\psi(x)=\frac{1}{\sqrt{\pi}a^{3/2}}e^{-r/a}$$

Hydrogen Atom

The hydrogen atom's electron, in the ground state, can exist at all radii (except 0).

$$P(r) = 4\pi r^2 |\psi(x)|^2 = \frac{4}{a^3} r^2 e^{-2r/a}$$



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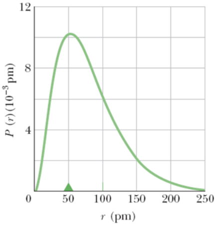
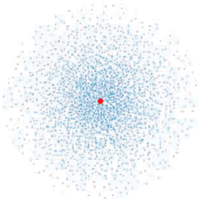
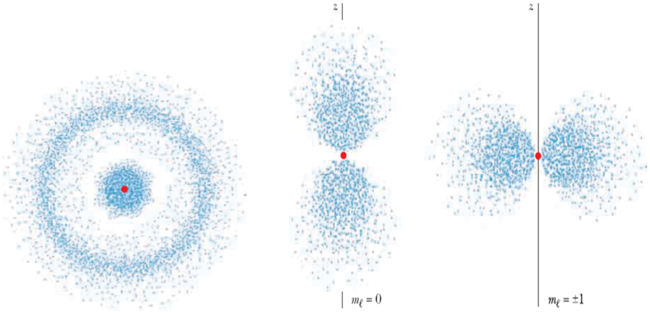




The peak of this function is at $r = a$, the Bohr radius!

Hydrogen Atom

Higher energy states of the hydrogen atom get more complex due to the quantized angular momentum.



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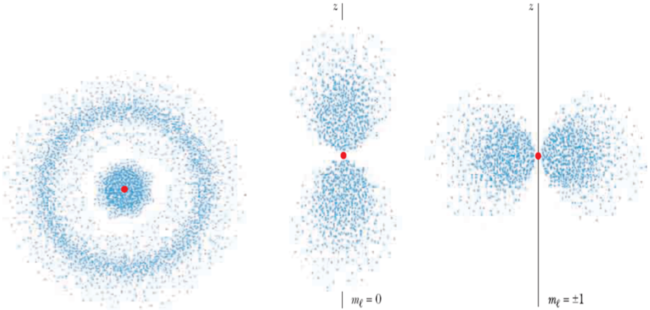

These states all have the same energy ($n = 2$) but different angular momenta ($l = 0, 1$).

Hydrogen Atom

There are three quantum numbers for the hydrogen atom.

Table 39-2

Quantum Numbers for the Hydrogen Atom

Symbol	Name	Allowed Values
n	Principal quantum number	$1, 2, 3, \ldots$
ℓ	Orbital quantum number	$0, 1, 2, \ldots, n-1$
m_ℓ	Orbital magnetic quantum number	$-\ell, -(\ell-1), \ldots, +(\ell-1), +\ell$

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Table 39-3

Quantum Numbers for Hydrogen Atom States with $n = 2$

n	ℓ	m_ℓ
2	0	0
2	1	+1
2	1	0
2	1	−1

Hydrogen Atom

Lecture Question 8.1

Which of the following most closely resembles the Bohr model of the hydrogen atom?

(a) A solid metal sphere with a net positive charge.

(b) A hollow metal sphere with a net negative charge.

(c) A tray full of mud with pebbles uniformly distributed throughout.

(d) The Moon orbiting the Earth.

(e) Two balls, one large and one small, connected by a spring.